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MODELS FOR COMPUTING THE DIRECTIONAL RADIATION OF SOUND FROM SOURCES ON A RIGID CYLINDRICAL BAFFLE

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Monterey, California

December 1974

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The closed-form equations describing the acoustic radiation pattern for a source flush-mounted on a rigid cylindrical baffle are derived for three sonar transducer design configurations: two rectangular designs (Segment and Patch) and one circular configuration (Disk). The derivation includes both application and an extension of developments by previous theorists. A computer program based on the derived closed-form

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(20. ABSTRACT Continued)

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Models for Computing the Directional
Radiation of Sound from Sources
on a Rigid Cylindrical Baffle

by

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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

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I. INTRODUCTION

Acoustic methods have for a long time proved useful for determining the position of objects submerged in the ocean and continue to be useful today, especially for monitoring the realistic testing of advanced underwater weapons (such as torpedoes) in the real ocean environment.

One method commonly used requires installation of a sound source on the underwater vehicle to be tracked. The source/vehicle is then tracked acoustically by hydrophones arrayed in a fixed position on the ocean bottom. The basic function is that of measuring the transit time of the sound wave from the source to different hydrophones. These transit times enable determination of distances and directions of arrival which in turn are triangulated to determine position, a series of which defines the vehicle's track.

As in most engineering problems, the requirements placed on an acoustic source used for tracking are often conflicting in nature, with the end result being an engineering compromise. For example, in order to achieve a broad pattern from a single element, the element's dimensions must be small relative to the wavelength of the sound transmitted. However, to obtain the desired sound pressure level, the element may have to be driven so hard that cavitation (an undesirable effect) results. For many practical reasons,

therefore, it is essential that the directional characteristics of a proposed transducer design be predicted prior to construction rather than measured afterwards. That is, a clear understanding of the relations between directivity and design constants is essential to properly design a transducer for a specific purpose.

Study of the geometry involved in tracking an underwater vehicle with ocean-floor mounted hydrophones (assuming that the hydrophones are not located at excessive depths) reveals that the slant range from the vehicle to the receivers is normally several orders of magnitude greater than the distance from the vehicle to the ocean floor. It is clear, then, that most of the acoustic energy from a source mounted on the underbody of a vehicle should be directed obliquely, with a relative minimum being transmitted directly down at the ocean floor. Consideration of a large slant range situation dictates that the directionality pattern extend almost to the horizontal, however, not to the degree that will result in surface reflections.

A feeling for the difficulty involved in practically achieving the radiation pattern previously described can be obtained by studying the P.M. Morse [Ref. 2] solution for a radially vibrating strip which extends indefinitely along a cylinder. For the physical dimensions of the baffle (torpedo) and the frequency (75KHz) involved in our case of interest, the Morse solution indicates the acoustic

radiation pattern will be highly directional and primarily will ensonify an area directly beneath the vehicle. That is, the combined effect of the cylindrical baffle and relatively high frequency focuses the acoustic radiation in a highly directional beam normal to the surface of the radiator. Obviously then, it can be anticipated that to achieve the obliquely oriented pattern desired, some method of countering this focusing must be incorporated in any proposed transducer design.

The intent of this report then is to describe procedures for computing one characteristic of a sound source used for underwater acoustic tracking - its acoustic radiation pattern. The approach will be mathematical in scope resulting in the development of a computer program which will calculate and plot the radiation pattern of a flush mounted radiator located on the wall of a cylindrical shape, such as that of a torpedo. The study will encompass three (3) proposed design configurations, described in the following section.

III. THEORY

A. BACKGROUND

The subject of sound radiation from vibrating objects is an old one and the journal literature is replete with papers on the theory. It appears that the particular problem at hand, radiation from a finite element on a rigid cylindrical baffle has not received much attention. Morse and Ingard [Ref. 3] treat the problem of radiation from line sources on a cylinder. Laird and Cohen [Ref. 1] developed a solution for the far field radiation from a rectangular patch on a rigid cylinder, which comes closest to representing the present problem. For this reason, it forms the basis for the model of rectangular sources and the beginning point for the model of a circular source. For the convenience of the reader, some of the results of Laird and Cohen are summarized below.

In their study, Laird and Cohen extended the theory of Morse [Ref. 2] to the case where the source is any separable function of the azimuthal and axial dimensions. In this thesis, their results will be applied to derive the equations for the "Patch" and "Segment" sources. In addition, an extension of their work to the general case of a non-separable source is included to describe the pattern of the "Disk" configuration.

The general approach developed by Laird and Cohen uses the cylindrical coordinate system shown in Figure 1. The method assumes that the source, which is mounted on a rigid cylinder, vibrates radially in such a manner that its velocity distribution may be represented as a separable function of the azimuth and axial dimensions. That is, the velocity distribution has the same functional form in the azimuthal direction independent of the axial dimension and vice versa. This is shown diagrammatically in Figure 2. The boundary condition at the surface of the cylinder/source can then be given by the following expression:

$$u_r|_{r=a} = U_o\, e^{-i\omega t} \left(\sum_{m=0}^{\infty} a_m \cos m\phi \right) \left[\int_{-\infty}^{\infty} F(k_z) e^{ik_z z} dk_z \right] \tag{1}$$

where the Fourier cosine series represents the azimuthal dependence and the Fourier integral represents the axial dependence. The cosine series was arbitrarily chosen for mathematical convenience.

Having established the boundary condition, the general expression for a combination of outgoing cylindrical waves of even azimuthal dependence, given by

$$p(r,\phi,z) = e^{-i\omega t} \sum_{m=0}^{\infty} \cos m\phi \times \int_{-\infty}^{\infty} A_m(k_z) H_m^{(1)}(k_r r) e^{ik_z z} dk_z \tag{2}$$

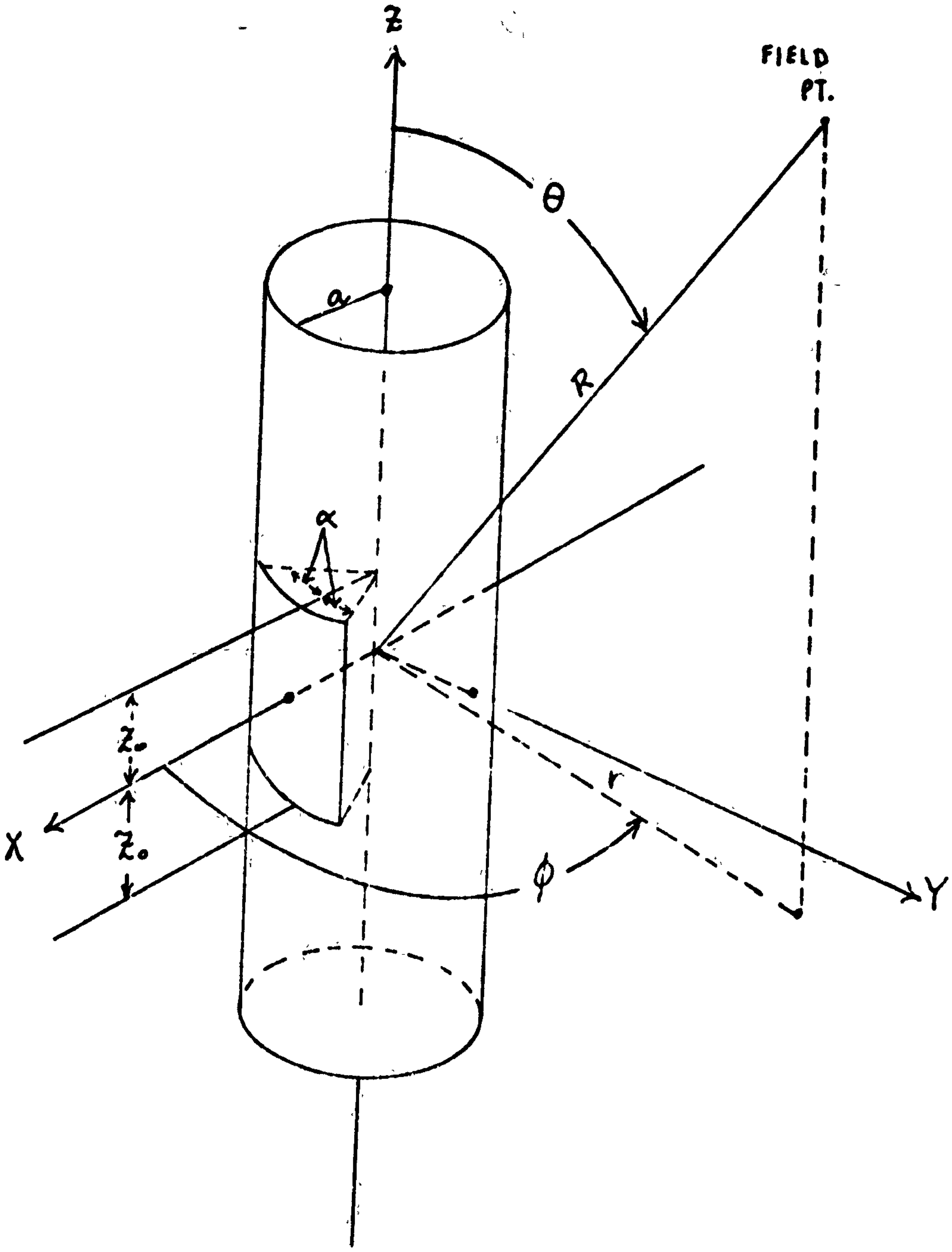


FIG. 1

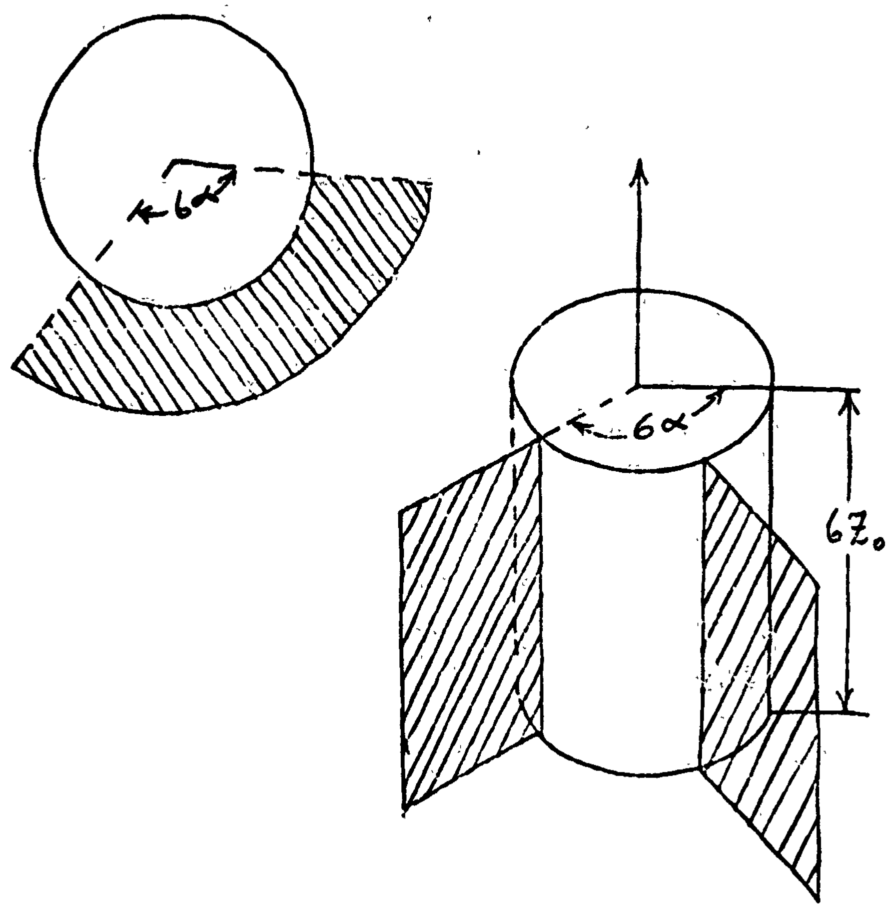


FIG. 2

is matched to the boundary condition at $r = a$ (the surface of the cylinder). Introduction of the far field approximation for the Hankel function, conversion to spherical coordinates, and the solution of a Fourier integral by the method of stationary phase result in the following general expression in spherical coordinates at a field point describing the radiation from a source on a rigid cylinder:

$$p(R,\theta,\phi) = 2\rho c U_o \frac{e^{i(kR-\omega t)}}{R} \frac{F(k_z)}{\sin\theta} \times \sum_{m=0}^{\infty} \frac{a_m e^{-im\frac{\pi}{2}}}{H_m^{\prime(1)}(ka\sin\theta)} \cos m\phi \tag{3}$$

To consider a specific source using this method, one needs only to specify its location on the cylinder, physical dimensions and velocity distribution. Knowing the above, the Fourier coefficients, a_m, describing the azimuthal dependence and the Fourier transform, $F(k_z)$, describing the axial dependence, can be calculated. Substitution of the Fourier coefficients and the Fourier transform into Equation (3) results in an expression describing the radiation for the particular source considered. The frequency dependence is incorporated through the wave number, "k".

The derivation for the "Patch" configuration parallels the development for the case of the uniform rectangular source calculated by Laird and Cohen and illustrates use of Equation (3) for determining the radiation from a specific separable source.

B. PATCH CONFIGURATION

The "Patch" array (see Figure 3) is composed of nine equal-dimensioned elements (each of angular width 2α and height $2Z_o$) with the center element 180° out of phase with respect to the other elements.

Consideration of Figure 4 shows that the source motions of the patch cannot be described by separable, independent functions of the cylindrical coordinates ϕ and Z.

To avoid the complexities involved with a non-separable source, the entire array will be viewed as a simple rectangular source with uniform distribution (see Figure 5).

Likewise, the center element will be considered as a simple, uniformly excited rectangular source with dimensions 2α by $2Z_o$.

By subtracting twice the pattern function of the center element from that of the entire nine-element array, the desired result is achieved. This assumes the validity of the linear superposition principle.

The coefficients associated with the functions describing the source motion in azimuth are calculated using standard Fourier Series relationships. This results in:

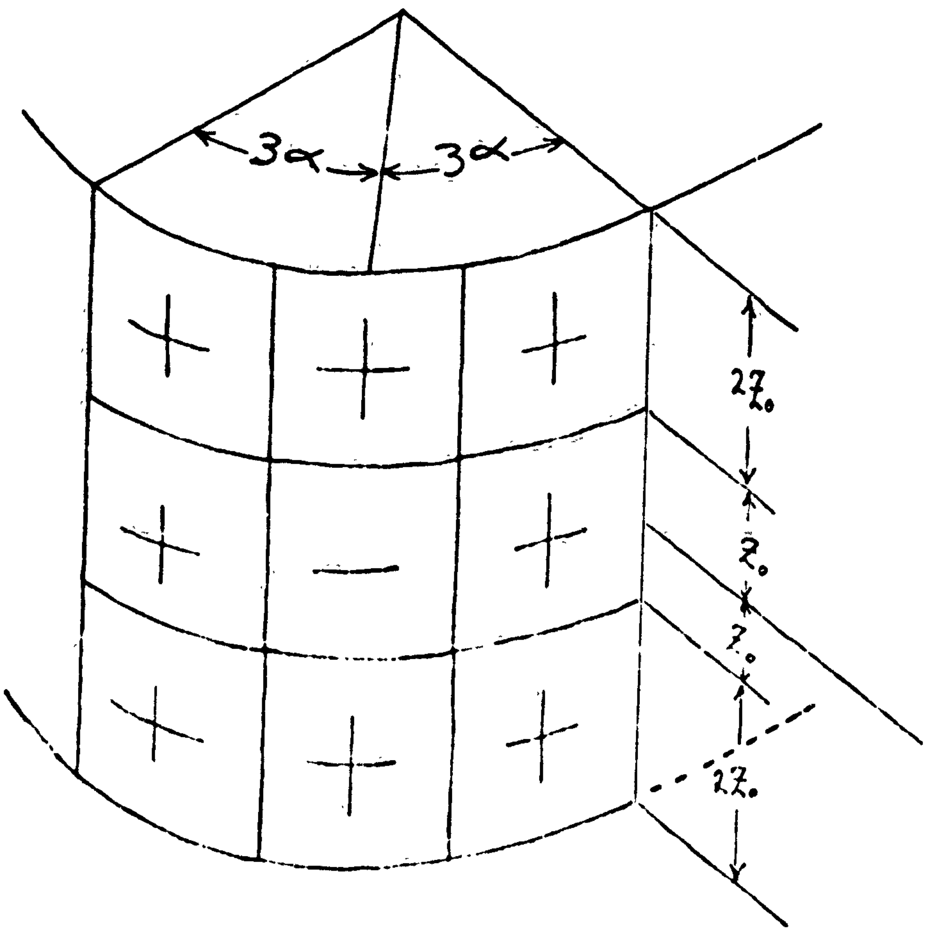


FIG. 3

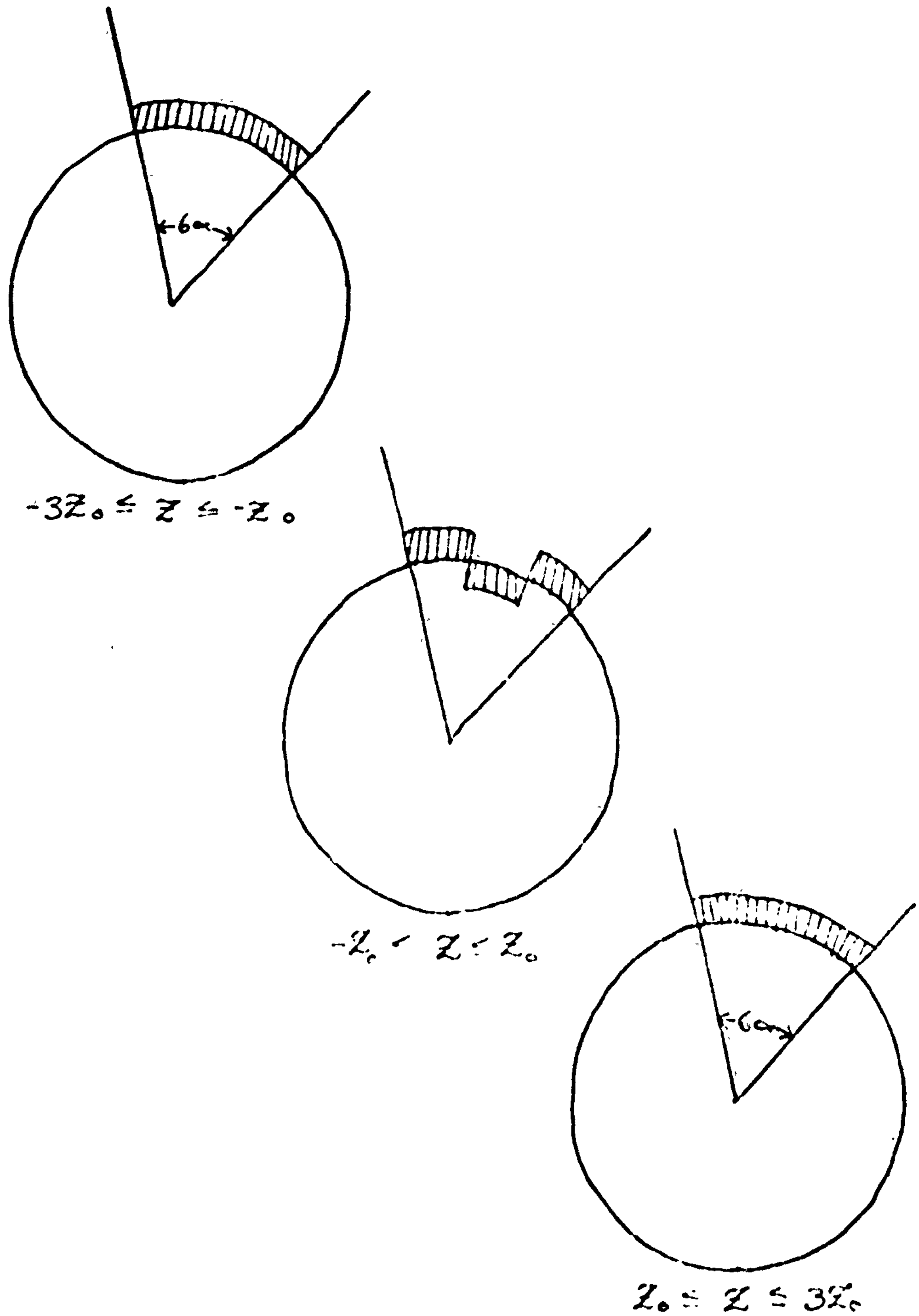


FIG. 4

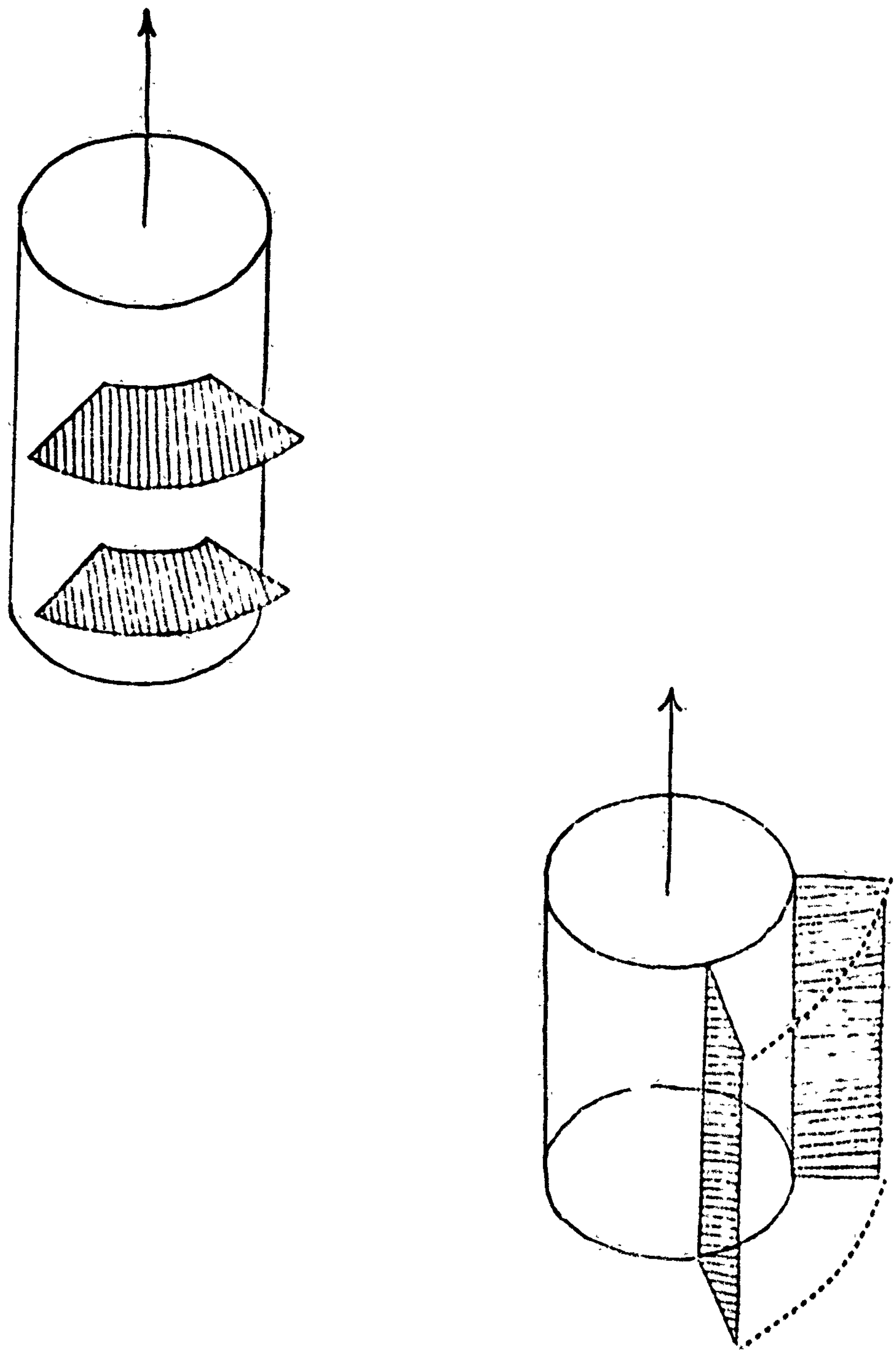


FIG. 5

For the center element of dimensions $(2\alpha, 2Z_o)$

$$a_o = \frac{\alpha}{\pi} \qquad a_m = \frac{2\sin m\alpha}{m\pi}$$

$$m = 1,2,3,\ldots$$

For the entire array of dimensions $(6\alpha, 6Z_o)$

$$a_o = \frac{3\alpha}{\pi} \qquad a_m = \frac{2\sin 3m\alpha}{m\pi}$$

$$m = 1,2,3,\ldots$$

Similarly the Fourier Transforms describing the axial dependence are

For the center element of dimensions $(2\alpha, 2Z_o)$

$$F(k_z) = \frac{\sin k_z Z_o}{\pi k_z}$$

For the entire array of dimensions $(6\alpha, 6Z_o)$

$$F(k_z) = \frac{\sin 3k_z Z_o}{\pi k_z}$$

Substitution into Equation (3) yields the following expression for the far-field pressure due to the radiation:

$$
\begin{aligned}
p(R,\theta,\phi) = 2\rho c U_o \frac{e^{i(kR-\omega t)}}{R} \Bigg[& \frac{\sin 3k_z Z_o}{\pi k_z} \Bigg(\frac{3\alpha/\pi}{H_o'^{(1)}(ka\sin\theta)} \\
& + \sum_{m=1}^{\infty} \frac{2\sin 3m\alpha/m\pi \left(e^{-im\frac{\pi}{2}}\right)}{H_m'^{(1)}(ka\sin\theta)} \cos m\phi \Bigg) \\
& - \frac{2\sin k_z Z_o}{\pi k_z} \Bigg(\frac{\alpha/\pi}{H_o'^{(1)}(ka\sin\theta)} \\
& + \sum_{m=1}^{\infty} \frac{2\sin m\alpha/m\pi \left(e^{-im\frac{\pi}{2}}\right)}{H_m'^{(1)}(ka\sin\theta)} \cos m\phi \Bigg) \Bigg] \qquad (4)
\end{aligned}
$$

C. SEGMENT [illegible]

The "Segment" array consists of elements of the same dimensions equally spaced about the circumference of the cylinder in the horizontal plane, that is, the plane perpendicular to the axis of the cylinder. This model permits specification of the amplitude and phase of motion for each element.

The following equation [Ref. 1] describes the far-field pressure in the horizontal plane for a single element of

dimensions 2α by $2Z_o$:

$$P(R,\tfrac{\pi}{2},\phi) = \frac{2\rho c U_o Z_o}{\pi} \frac{e^{i(kR-\omega t)}}{R} \times \left(\frac{\alpha/\pi}{H_o'^{(1)}(ka)} + \sum_{m=1}^{\infty} \frac{2\sin m\alpha/m\pi \left(e^{-im\frac{\pi}{2}}\right)}{H_m'^{(1)}(ka)} \cos m\phi\right) \tag{5}$$

In order to sum the contributions from all other elements which are disposed at uniform angles around the cylinder, it is necessary to transform the angle (ϕ) in Equation (5), so that it represents ϕ_i, the relative angle from the ith element to the field point, where the field point has spherical coordinates $(R,\frac{\pi}{2},\beta)$ measured from the center of the cylinder.

Figure 6 illustrates the geometry of this for the ith element. The total number of elements was arbitrarily chosen to be 120. Thus, the spacing between adjacent elements is three (3) degrees for the computations for this design. The angular position of element number one (1) was arbitrarily chosen to be located at $\beta = 0$. The sum of the radiation from all elements, the pattern of each of which is given by Equation (5), results in:

$$P(R,\tfrac{\pi}{2},\beta) = \frac{2\rho c U_o Z_o}{\pi} \frac{e^{i(kR-\omega t)}}{R} \sum_{i=1}^{120} A_i\left(e^{i\gamma_i}\right)\left[\frac{\alpha/\pi}{H_o'^{(1)}(ka)} + \sum_{m=1}^{\infty} \frac{2\sin m\alpha/m\pi \left(e^{-im\frac{\pi}{2}}\right)}{H_m'^{(1)}(ka)} \cos m\phi_i\right] \tag{6}$$

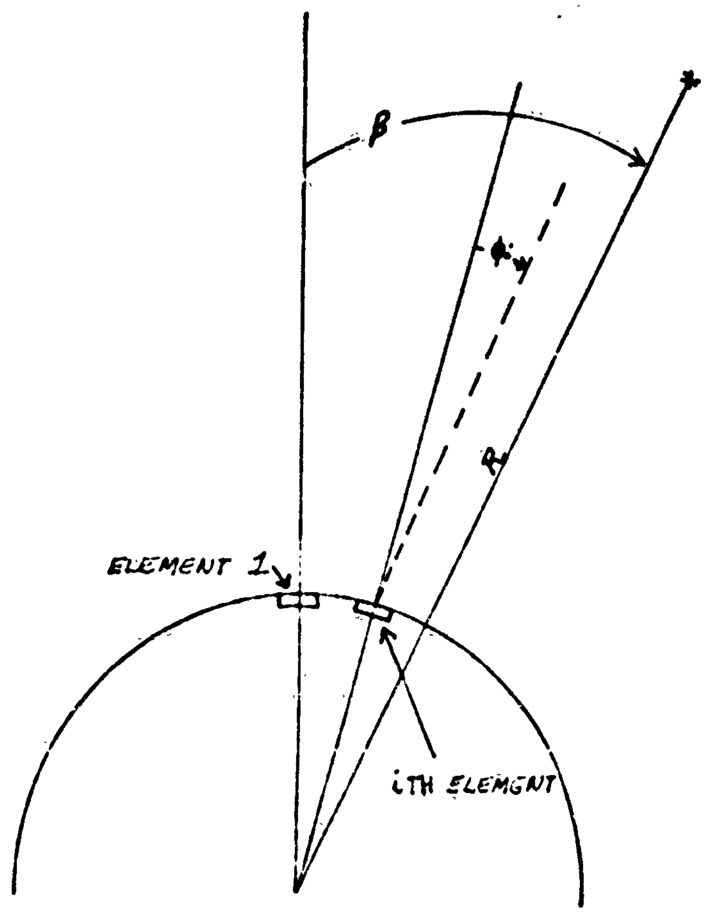


FIG. 6

where A_i is the relative amplitude of the ith element and γ_i is its relative phase and $\phi_i = [(i-1)3 + \beta]$.

In a similar manner, the following result is obtained for pressure as a function of the angle θ in a plane containing the axis of the cylinder:

$$P(R,\theta,\beta) = \frac{2\rho c U_c}{\pi} \frac{e^{i(kR-\omega t)}}{R} \frac{\sin(kZ_o\cos\theta)}{k\cos\theta\,\sin\theta} \times$$

$$\sum_{i=1}^{120} A_i e^{i\gamma_i}\left[\frac{\alpha/\pi}{H_o'^{(1)}(ka\sin\theta)} + \sum_{m=1}^{\infty} \frac{2\sin m\alpha/m\pi\,(e^{-im\frac{\pi}{2}})}{H_m'^{(1)}(ka\sin\theta)}\cos m\phi_i\right] \tag{7}$$

where $\phi_i = [(i-1)3 + \beta]$, "i" = the ith element number.

D. CIRCULAR PISTON SOURCE (DISK)

The "Disk" array is composed of a central circular piston source which is surrounded by a concentric annular piston, with a 180 degree phase difference between their motions.

Since the motions of a circular piston located on the side of a cylinder cannot be described as separable functions of the cylinder coordinates, an extension of the methods described above must be made. Figure 7 shows (assuming a uniform velocity distribution) that although the functional form of the velocity distribution remains the same across the disk, the source is non-separable in that the axial limits are dependent on the azimuth dimension and vice versa.

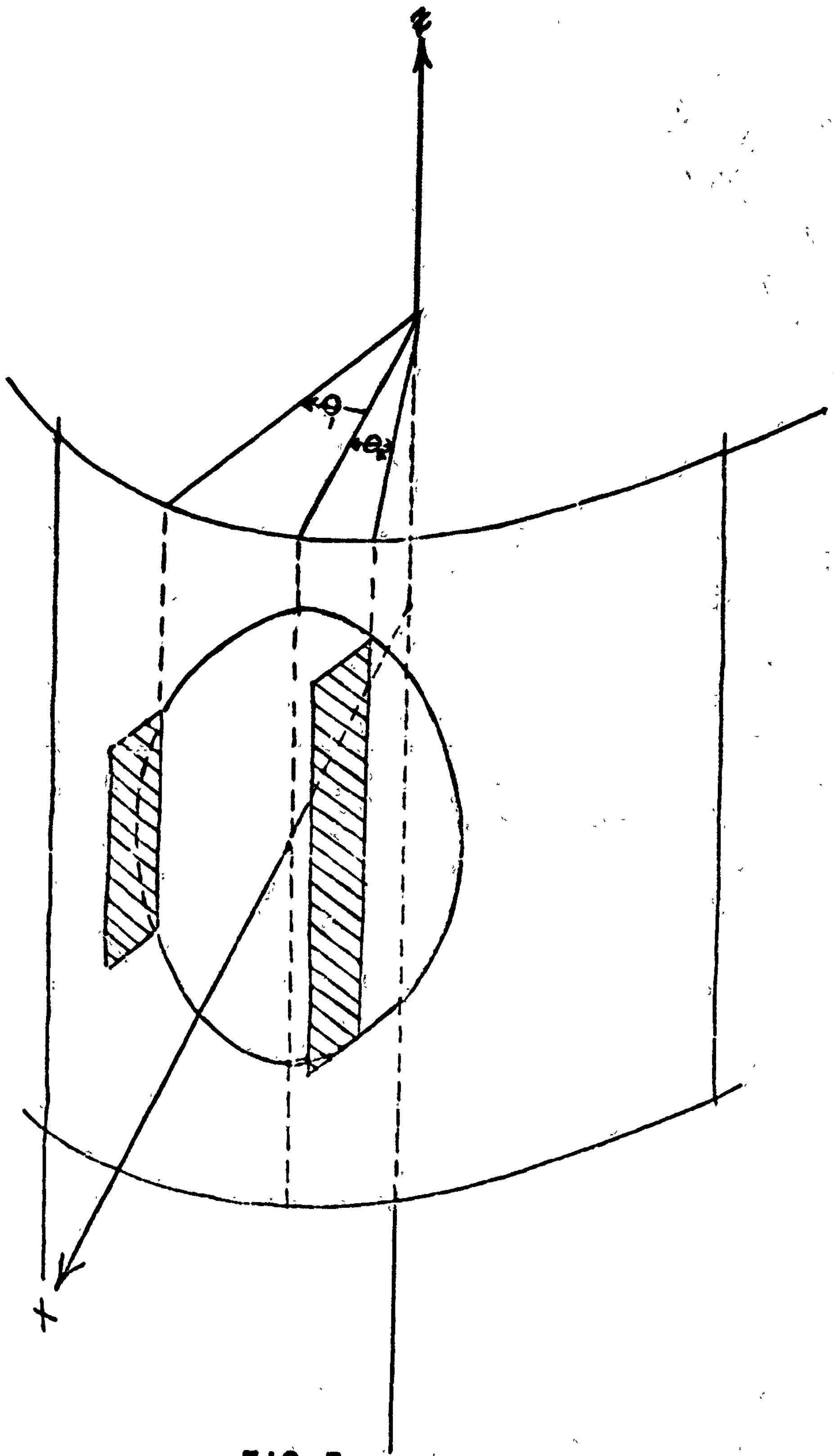


FIG. 7

Applying the boundary conditions for the circular piston using Equations (1) and (2) and the basic relation between particle velocity and pressure

$$u_r = -\frac{i}{\omega\rho}\frac{\partial p}{\partial r} \tag{8}$$

results in:

$$u_{r|r=a} = -\frac{i}{\rho\omega} e^{-i\omega t} \sum_{m=0}^{\infty} (\cos m\phi) \int_{-\infty}^{+\infty} k_r A_m(k_z) H_m'^{(1)} (k_r a) e^{ik_z Z} dk_z \tag{9}$$

Equating Equations (1) and (9) permits solution for $A_m(k_z)$ in terms of the Fourier coefficients, a_m. In the case of the separable source, the Fourier coefficients, a_m , are functions of the azimuth dimension of the source, ϕ. However, in the case of the non-separable source, they are also functions of the axial dimension (Z) and as such are expressible themselves as Fourier integrals.

To complete the general solution; having solved for $A_m(k_z)$, (in the separable case, $A_m(k_z) = \frac{i\rho\omega U_o a_m F(k_z)}{k_r H_m'^{(1)} (k_r a)}$) we substitute into Equation (2), the general equation for outgoing cylindrical waves of even azimuth dependence which yields the following:

$$p(r,\phi,Z) = e^{-i\omega t} \sum_{m=0}^{\infty} \cos m\phi \int_{-\infty}^{+\infty} \frac{i\rho\omega U_o a_m}{k_r H_m'^{(1)} (k_r a)} F(k_z) (H_m^{(1)} (k_r r) e^{ik_z Z} dk_z) \tag{10}$$

Introducing the far-field approximation for the Hankel function

$$H_m^{(1)}(k_r r) = \sqrt{\frac{2}{\pi k_r r}} \; e^{ik_r r} \; e^{-i(\frac{m\pi}{2} + \frac{\pi}{4})} \tag{11}$$

results in:

$$p(r,\phi,Z) = i\omega\rho U_o \sqrt{\frac{2}{\pi r}} \; e^{-i\omega t} \sum_{m=0}^{\infty} a_m \cos m\phi \, e^{-i(m+\frac{1}{2})\frac{\pi}{2}}$$

$$\left(\int_{-\infty}^{+\infty} \frac{F(k_z) e^{i(k_r r + k_z z)} \, dk_z}{k_r^{\frac{3}{2}} H_m'^{(1)} \; (k_r a)} \right) . \tag{12}$$

Converting to spherical coordinates, results in:

$$p(r,\theta,Z) = i\omega\rho U_o \left(\frac{2}{\pi R \sin\theta}\right)^{\frac{1}{2}} e^{-i\omega t}$$

$$\times \sum_{m=0}^{\infty} a_m e^{-i(m+\frac{1}{2})\frac{\pi}{2}} \cos m\phi \tag{13}$$

$$\times \int_{-\infty}^{+\infty} \frac{F(k_z) \exp\{i[(k^2-k_z^2)^{\frac{1}{2}} R\sin\theta + k_z R\cos\theta)]\}}{(k^2-k_z^2)^{\frac{3}{4}} H_m'^{(1)} \; [(k^2-k_z^2)^{\frac{1}{2}} a]} \, dk_z$$

Evaluating the Fourier Integral by the method of stationary phase gives as a final expression for the radiation:

$$p(R,\theta,\phi) = 2\rho c U_o \frac{e^{i(kR-\omega t)}}{R} \frac{F(k_z)}{\sin\theta} \times \sum_{m=0}^{\infty} \frac{a_m e^{-im\frac{\pi}{2}}}{H_m'^{(1)} \; (ka\sin\theta)} \cos m\phi \tag{14}$$

Returning to the non-separable disk, the boundary condition Equation (1), is again expressed as:

$$u_{r|r=a} = U_o e^{-i\omega t} [\sum_{m=0}^{\infty} \cos m\phi \; a_m(z)] f(z) \qquad (15)$$

where

$$f(z) = \int_{-\infty}^{\infty} F(k_z) e^{ik_z z} \, dk_z .$$

Since each term of the sum over "m" is multiplied by f(z), we may include f(z) in the sum. Thus,

$$u_{r|r=a} = U_o e^{-i\omega t} \sum_{m=0}^{\infty} \cos m\phi \; a_m(z) f(z) \qquad (16)$$

Now, let $g_m(z) = a_m(z) f(z)$.

Define

$$G_m(k_z) = F\{g_m(z)\} \text{ just as } F(k_z) = F\{f(z)\}$$

Then Equation (16) becomes:

$$u_{r|r=a} = U_o e^{-i\omega t} \sum_{m=0}^{\infty} \cos m\phi \int_{-\infty}^{+\infty} G_m(k_z) e^{ik_z z} \, dk_z \qquad (17)$$

Let $a_m = 1$ in Equation (1). Then Equation (17) is identical to Equation (1), which is from Laird and Cohen's development except that our $G_m(k_z)$ depends on "m" while the $F(k_z)$ in

Equation (3) does not. However, this still permits using Laird and Cohen's results; namely Equation (13) still applies with a_m in (13) set equal to one (1) and $G_m(k_z)$ replacing $F(k_z)$.

The parallel to Equation (14) then is:

$$p(R,\theta,\phi) = 2\rho c U_o \frac{e^{i(kR-\omega t)}}{R \sin\theta} \sum_{m=0}^{\infty} \frac{G_m(k_z)e^{-im\frac{\pi}{2}}}{H_m'^{(1)}(ka\sin\theta)} \cos m\phi \qquad (18)$$

where we have set a_m equal to one (1) in Equation (14) and have replaced $F(k_z)$ by $G_m(k_z)$, but have included $G_m(k_z)$ within the summation over "m" because $G_m(k_z)$, unlike $F(k_z)$, depends on "m".

It remains, then, to evaluate $G_m(k_z)$ for the case at hand. The following was accomplished in collaboration with Steven R. Cohen [Ref. 5].

Noting that the equation of the disk on the cylinder is given by $z^2 + a^2\phi^2 = z_o^2$ where $z_o = a\alpha$, we solve for "ϕ" in terms of "z"

$$\phi = \pm\sqrt{\frac{z_o^2 - z^2}{a^2}} \quad .$$

Using this result, we can express the Fourier coefficients directly in terms of (z), the axial dimension:

$$a_o = \frac{\phi}{\pi} = \sqrt{\frac{z_o^2 - z^2}{a^2}}\left(\frac{1}{\pi}\right)$$

$$a_m = 2\sin\left(\frac{m\sqrt{\frac{z_o^2 - z^2}{a^2}}}{m\pi}\right) = \frac{2\sin\left(\frac{m}{a}\sqrt{z_o^2 - z^2}\right)}{m\pi} \qquad (19)$$

Since $f(z) = 1$ (a constant)

$$G_m(k_z) = F\{a_m(z)f(z)\}$$

$$= F\{a_m(z)\} = F\{\frac{2}{m\pi} \sin[\frac{m}{a}(z_o{}^2 - z^2)^{\frac{1}{2}}]\}$$

$$= \frac{2}{m\pi} \int_{-\infty}^{+\infty} \frac{e^{i\frac{m}{a}\sqrt{z_o{}^2-z^2}} - e^{-i\frac{m}{a}\sqrt{z_o{}^2-z^2}}}{2i} e^{-ik_z z}\, dz \quad (20)$$

To evaluate this integral, we first let $k_z = \omega$, $z = t$ and $dz = dt$. Therefore

$$G_m(k_z) = \frac{1}{im\pi} \int_{-\infty}^{+\infty} (e^{i\frac{m}{a}\sqrt{z_o{}^2-t^2}} - e^{-i\frac{m}{a}\sqrt{z_o{}^2-t^2}})e^{-i\omega t}\, dt \quad (21)$$

that is:

$$G_m(\omega) = F\{\frac{1}{m\pi i} e^{i\frac{m}{a}\sqrt{z_o{}^2-t^2}}\} - F\{\frac{1}{m\pi i} e^{-i\frac{m}{a}\sqrt{z_o{}^2-t^2}}\} \quad (22)$$

To evaluate the above Fourier transforms we will use the following standard Laplace transform relationships:

(1) $L\{J_o(a\sqrt{t^2-b^2})\} = \dfrac{e^{-b\sqrt{s^2-a^2}}}{\sqrt{s^2+a^2}}$ for $t > b$

(2) $L\{C(t,a)\} = C(s,a)$

(3) $\int L\{C(t,a)\}da = \int C(s,a)\, da$

Applying relationship (3) to (1)

$$L\{\int J_0(a\sqrt{t^2-b^2}\,da\} = \int \frac{e^{-b\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}\,da \tag{23}$$

$$= -\frac{1}{ab}\,e^{-b\sqrt{s^2+a^2}} \quad .$$

Letting $b = \frac{im}{c}$, where "c" is chosen vice "a" to avoid confusion, i.e., c = a, and substituting into (23) yields

$$L\{\int J_0(a\sqrt{t^2+\frac{m^2}{c^2}}\,)da\} = -\frac{1}{a(\frac{im}{c})}\,e^{-\frac{im}{c}\sqrt{s^2+a^2}} \quad . \tag{24}$$

To evaluate the integral, $\int J_0(a\sqrt{t^2+\frac{m^2}{c^2}})\,da$:

Let $u = a\sqrt{t^2+\frac{m^2}{c^2}}$. Therefore $du = \sqrt{t^2+\frac{m^2}{c^2}}\,da$,

$$da = \frac{du}{\sqrt{t^2+\frac{m^2}{c^2}}}$$

$$\int J_0(a\sqrt{t^2+\frac{m^2}{c^2}}\,)da = \frac{1}{\sqrt{t^2+\frac{m^2}{c^2}}}\int J_0(u)\,du$$

$$= \frac{1}{\sqrt{t^2+\frac{m^2}{c^2}}}(2\sum_{m=0}^{\infty} J_{2m+1}(u))$$

$$= \frac{2}{\sqrt{t^2+\frac{m^2}{c^2}}}[J_1(u)+J_3(u)+J_5(u)+\ldots] \; . \tag{25}$$

Therefore Equation (24) becomes

$$L\{\frac{2}{\sqrt{t^2+\frac{m^2}{c^2}}}\,(J_1(a\sqrt{t^2+\frac{m^2}{c^2}}\,) + J_3(a\sqrt{t^2+\frac{m^2}{c^2}}\,) + J_5(\qquad) + \ldots]\}$$

$$= -\frac{c}{aim}\, e^{-\frac{im}{c}\sqrt{s^2+a^2}} \qquad (26)$$

Substituting $s = j\omega$ to convert to the Fourier variable

$$F\{\frac{2}{\sqrt{t^2+\frac{m^2}{c^2}}}\,(J_1(a\sqrt{t^2+\frac{m^2}{c^2}}\,) + J_3(a\sqrt{t^2+\frac{m^2}{c^2}}\,) + \ldots\,)\}$$

$$= -\frac{c}{aim}\, e^{-\frac{im}{c}\sqrt{a^2-\omega^2}} \qquad (27)$$

which is of the form $F\{f(t)\} = F(\omega)$.

Using the relationship $F\{F(t)\} = 2\pi f(-\omega)$ with Equation (27) gives the result:

$$F\{-\frac{c}{aim}\, e^{-\frac{im}{c}\sqrt{a^2-t^2}}\,\}$$

$$= \frac{4\pi}{\sqrt{\omega^2+\frac{m^2}{c^2}}}\,[J_1(a\sqrt{\omega^2+\frac{m^2}{c^2}}) + J_3(a\sqrt{\omega^2+\frac{m^2}{c^2}}) + J_5(\qquad) + \ldots] \qquad (28)$$

Taking the constant $(-\frac{c}{aim})$ outside the transform and dividing gives:

$$F\{e^{-\frac{im}{c}\sqrt{a^2-t^2}}\}$$

$$= -\frac{aim}{c} \cdot \frac{4\pi}{\sqrt{\omega^2+\frac{m^2}{c^2}}} [J_1(a\sqrt{\omega^2+\frac{m^2}{c^2}}) + J_3(a\sqrt{\omega^2+\frac{m^2}{c^2}}) + J_5(\quad)+\ldots] \tag{29}$$

which can be directly used to evaluate

$$F\{-\frac{1}{m\pi i} e^{-\frac{im}{a}\sqrt{z_o^2-z^2}}\}$$

where m = m, c = a, and a = z_o. Therefore,

$$F\{-\frac{1}{m\pi i} e^{-\frac{im}{a}\sqrt{z_o^2-z^2}}\}$$

$$= -\frac{1}{m\pi i} \cdot \frac{miz_o}{a} \cdot \frac{4\pi}{\sqrt{\omega^2+\frac{m^2}{a^2}}} [J_1(z_o\sqrt{\omega^2+\frac{m^2}{a^2}} + J_3(\quad) + \ldots]$$

$$= \frac{4\pi z_o}{a\sqrt{\omega^2+\frac{m^2}{a^2}}} [J_1(z_o\sqrt{\omega^2+\frac{m^2}{a^2}}) + J_3(\quad) + J_5(\quad) + \ldots] \tag{30}$$

A similar treatment letting b = $\frac{im}{c}$ permits the following:

$$F\{\frac{1}{m\pi i} e^{-\frac{im}{a}\sqrt{z_o^2-z^2}}$$

$$= \frac{4z_o}{a\sqrt{\omega^2+\frac{m^2}{a^2}}} [J_1(z_o\sqrt{\omega^2+\frac{m^2}{a^2}}) + J_3(\quad) + J_5(\quad) + \ldots] \tag{31}$$

Since

$$G_m(k_z) = F\{\frac{2}{m\pi} \sin(\frac{m}{a}\sqrt{z_o^2 - z^2}\ )\ \}$$

$$= F\{\frac{1}{m\pi i} \cdot e^{\frac{im}{a}\sqrt{z_o^2-z^2}}\} + F\{-\frac{1}{m\pi i}\ e^{-\frac{im}{a}\sqrt{z_o^2-z^2}}\}$$

$$G_m(k_z) = \frac{8z_o}{a} \cdot \frac{1}{\sqrt{k_z^2+\frac{m^2}{a^2}}} [J_1(z_o\sqrt{k_z^2+\frac{m^2}{a^2}}\ ) + J_3(z_o\sqrt{k_z^2+\frac{m^2}{a^2}}\ )$$

$$+ J_5(\quad) + \ldots]$$

$$= \frac{8z_o^2}{a} [\frac{J_1(z_o\sqrt{k_z^2+\frac{m^2}{a^2}}\ )}{z_o\sqrt{k_z^2+\frac{m^2}{a^2}}} + \frac{J_3(z_o\sqrt{k_z^2+\frac{m^2}{a^2}}\ )}{z_o\sqrt{k_z^2+\frac{m^2}{a^2}}}$$

$$+ \frac{J_5(\quad)}{(\quad)} + \ldots\ ] \tag{32}$$

Substituting $G_m(k_z)$ into the general expression for the radiation, Equation (19), results in the following equation describing the radiation for a "disk" source on a cylindrical baffle:

$$p(R,\theta,\phi) = \frac{16\rho c U_o z_o^2}{a} \frac{e^{i(kR-\omega t)}}{R\sin\theta} \sum_{m=0}^{\infty} \sum_{i=0}^{\infty} \frac{J_{2i+1}(z_o\sqrt{k_z^2+\frac{m^2}{a^2}}\ )}{z_o\sqrt{k_z^2+\frac{m^2}{a^2}}}$$

$$\times \frac{e^{-im\frac{\pi}{2}}}{H_m'^{(1)}(ka\sin\theta)} \cos m\phi \tag{33}$$

The pattern for the "Disk" design is achieved by subtracting twice the pattern of the inner circular piston source from that of a circular piston source having a diameter equal to that of the outer concentric annular piston.

III. COMPUTER MODELS

A. GENERAL DESCRIPTION

Based on the closed-form equations derived in the previous section, a computer program was written to permit rapid calculation and plotting of the directivity patterns for the designs of interest. A complete listing of the program developed is included as Appendix A. A brief description of each of the major subroutines which comprise the program is included as Appendix B.

Through the use of the program, the parameters which effect the radiation pattern of a particular source can be varied and their effect on the radiation pattern noted. In this manner, a design for a particular configuration can be achieved.

The parameters affecting the radiation pattern are changed through the use of "data inputs" to the program. The data inputs permit the following: (1) changing the physical dimensions of each of the sources, (2) changing the frequency at which the source is operated, (3) changing the radius of the cylindrical baffle on which the source is mounted, and (4) selecting the plane in which the directivity pattern is desired. In addition, for the use of the "Segment" design, the program inputs permit specifying the amplitude and phase shading of each of the individual

elements of the array. However, in this case only the resulting beam pattern in the horizontal plane can be plotted.

Another input which was added to increase the flexibility of operation of the program is the "summations limit". As can be noted from the equations derived in the previous section, all of the radiation pattern equations include an infinite series summation. Although this limit remains an input variable to the program, judicious use of this input is recommended if correct results are to be obtained. This matter will be discussed further in the "Results" section of this thesis.

Program outputs are both tabular and graphical in character and can be summarized as follows: (1) The computed values of the real and imaginary parts of the complex pressure for one (1) degree bearing increments expressed in dynes per square meter, (2) The magnitude of the complex pressure for one (1) degree bearing increments expressed in dynes per square meter, (3) The magnitude of the complex pressure for one (1) degree bearing increments expressed in decibels referenced to one microbar, (4) A polar plot of the magnitude of the complex pressure normalized to the largest value of the complex pressure, (5) A polar plot of the sound pressure level in decibels, normalized to the level of the largest lobe, using a scale length of 50 dB.

B. DESCRIPTION OF COMPUTER PROGRAM OPERATING PROCEDURES

A sample input data deck is included as Appendix C to illustrate these written instructions.

With the exception of the amplitude and phase shading portion of the "Segment" design, the input data decks required to obtain directionality patterns for each of the three (3) configurations are identical. The first data card indicates the configuration for which a beam pattern is desired. This is accomplished by typing the name of the configuration (PATCH, DISK, OR SEGEMENT) as shown on the first data card commencing in column number one (1). The second data card indicates the number of summations desired and the physical characteristics of an <u>element</u> of the particular design for which the pattern is being calculated. The number of summations desired is typed in columns one (1) and two (2), [FORMAT I2], one-half the height (z_o) of a single element measured in meters is entered in columns 11-20 [FORMAT F10.5) (in the case of the DISK the outer radius measured in meters), one-half the angular width of a single element (α) measured in radians is entered in columns 21-30 [FORMAT F10.5] (in the case of the DISK - the inner radius measured in meters), the radius of the cylinder measured in meters in columns 31-40, and frequency in KHz (i.e. for 75 KHz type "75.") in columns 41-50 [FORMAT F10.5]. The third and final data card indicates the plane in which

a pattern is desired and the angle which is to be held constant for the calculation. By reference to Figure 1, one can see that by setting ϕ equal to a constant and summing θ, a pattern in a plane containing the axis of the cylinder is achieved. Likewise, by setting θ equal to a constant and summing ϕ, a pattern for a plane orthogonal to the axis of the cylinder is obtained. In this manner, various planes for a particular source and three (3) dimensional aspects of the pattern can be perceived and investigated. Indication of the angle which it is desired to vary is indicated by typing a "1" (Theta varying) or a "2" (Phi varying) in columns 1-2 [FORMAT I2]. The angle held constant is indicated by typing the angle (in degrees) in columns 11-20 [FORMAT F10.5].

To obtain a pattern in the horizontal plane for amplitude and phase shading for the "Segment" design, enter a "1" or "2" in column "2" [FORMAT I2] of the fourth card of the Segment Data Deck. On the subsequent card (the fifth card of the data deck) enter the total number of elements to be shaded using columns 1-3 [FORMAT I3] (i.e. if 24 elements are to be shaded, type "24" in columns 2 and 3). Following this card, a separate card for each of the elements requiring shading is entered. In columns 1-3 the element number to be shaded is entered [FORMAT I3], in columns 11-20 the amplitude of the shading is entered, [FORMAT F10.5] in columns 21-30 the desired phase shading is entered [FORMAT F10.5] as an angle.

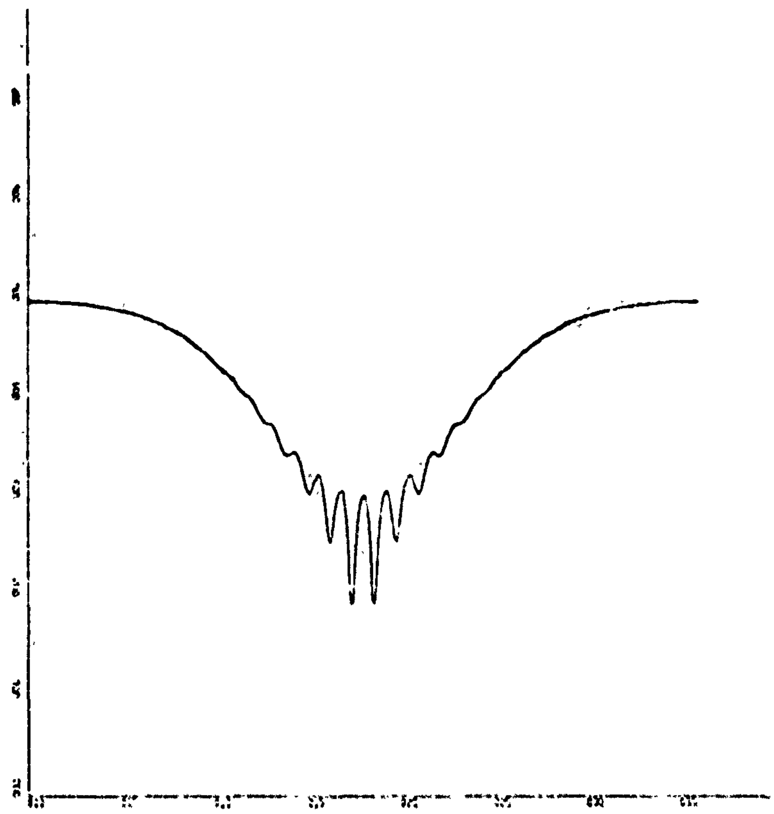
IV. RESULTS

A. PATTERN FOR UNIFORMLY EXCITED RECTANGULAR PATCH

Figure 8 shows the horizontal directivity pattern calculated for a single element ($\alpha = 3.7°$, ka = 14) using the "Segment" subroutine. The pattern was calculated and plotted in rectangular coordinates to permit a direct comparison to a similar pattern plotted by Laird and Cohen [Ref. 1] using the same input variables. Comparison of the two patterns shows they are identical. This result verifies that the basic coding of the program is correct.

B. EFFECT OF VARYING THE NUMBER OF SUMMATIONS

As noted previously, judicious use of the summing limit is required if accurate results are to be obtained. Runs for the same configuration were made with N = 5, 10, 15, 20, 25, 30, and 35. Little to no observable change was noted in patterns for the runs with "N" greater than twenty (20). However, for runs with "N" less than twenty (20), the pattern varied considerably for each run. Experience with the program indicates that about 20-25 terms of the infinite series (which appears in the pattern function of all designs considered) must be calculated before the divergence of the Hankel derivations in the denominator cause the solution to converge. This conclusion concurs with that arrived at by Laird and Cohen in their original development.



X-SCALE 5.00E+01 UNITS INCH.
Y-SCALE 1.00E+01 UNITS INCH.
PLOT OF RAD IN DB

FIG. 8

As indicated by the comments in the BESJ SUBROUTINE listing, a maximum of twenty summations should be calculated if the value of the entering argument is less than fifteen (15).

To meet both the previously mentioned requirements, it is recommended that the summing limit always be set equal to twenty (20) for any final computations of a proposed design. The ability to change the number of summations has been included to permit a rapid rough first-cut. The execution time for the computer solution varies from 5-20 minutes dependent on the number of summations and the input parameters.

C. FREQUENCY DEPENDENCE OF THE SOLUTION

Figures 9, 10, 11 and 12 are included as examples of the manner in which the frequency dependence of the patterns can be studied. All input variables were held constant and the frequency was varied to obtain these patterns. The important design capability provided by this feature is the ability to predict the effect of the frequency bandwidth of a proposed design on the acoustic radiation pattern.

THIS RUN IS FOR A TRANSDUCER OF THE SEGMENT TYPE.

NUMBER OF SUMMATIONS	20
TRANSDUCER HEIGHT (METERS)	.20000E-01
TRANSDUCER WIDTH(RADIANS)	.18748E-01
RADIUS OF CYLINDER(METERS)	.20670E 00
FREQUENCY(KHZ)	.65000E 02
TYPE OF PLOT (1) THETA VARYING, 2) PHI VARYING)	2
PLANE ANGLE HELD CONSTANT FOR PLOT	.90000E 02

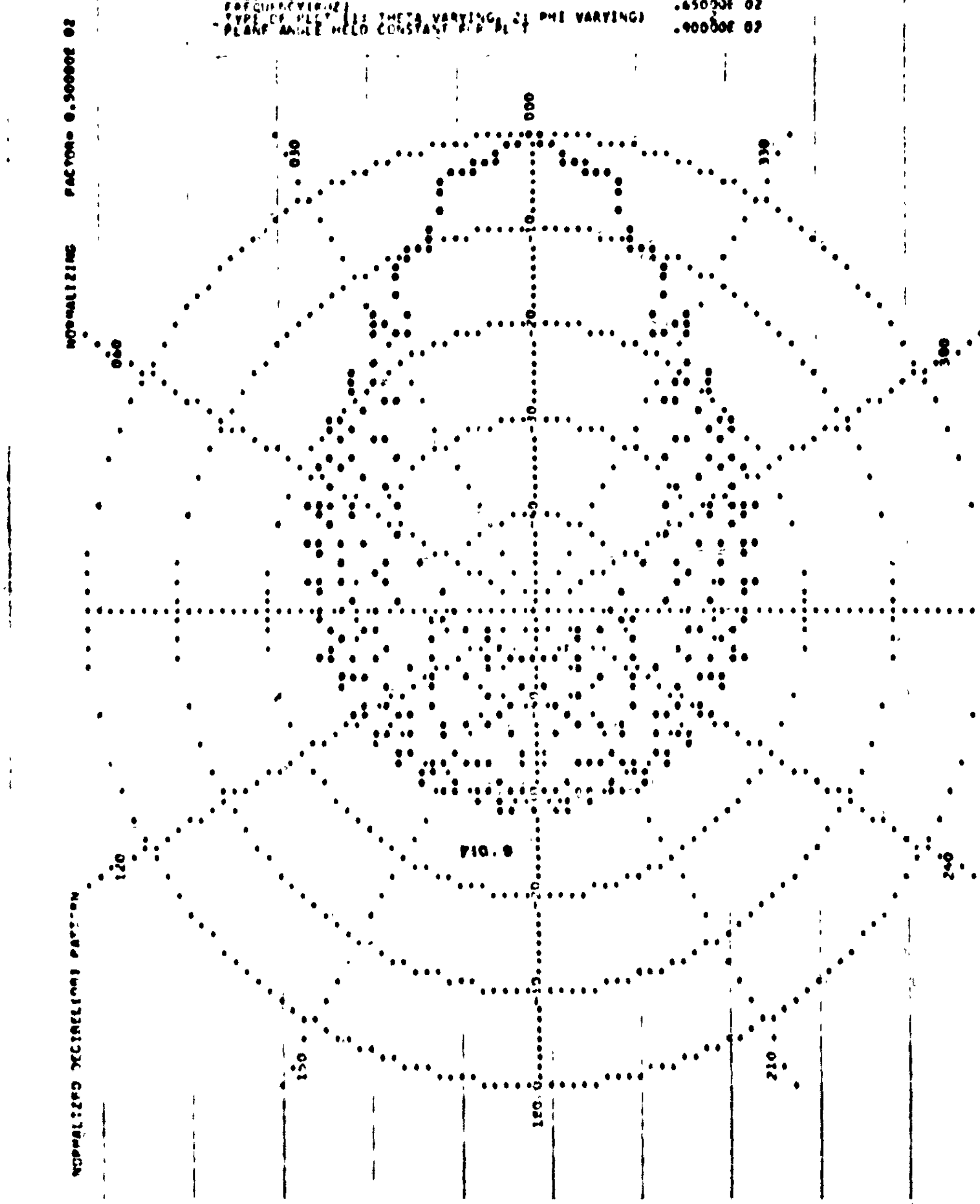


FIG. 8

THIS RUN IS FOR A TRANSDUCER OF THE SEGMENT TYPE.

NUMBER OF SUMMATIONS	20
TRANSDUCER HEIGHT (METERS)	.20000E-01
TRANSDUCER WIDTH (RADIANS)	.18749E-01
RADIUS OF CYLIN (METERS)	.26670E 00
FREQUENCY(KHZ)	.75000E 02
TYPE OF PLOT (1: THETA VARYING, 2: PHI VARYING)	2
PLANE ANGLE HELD CONSTANT FOR PLOT	.90000E 02

NORMALIZED RECTILINEAR PATTERN

NORMALIZING FACTOR= 0.50000E 02

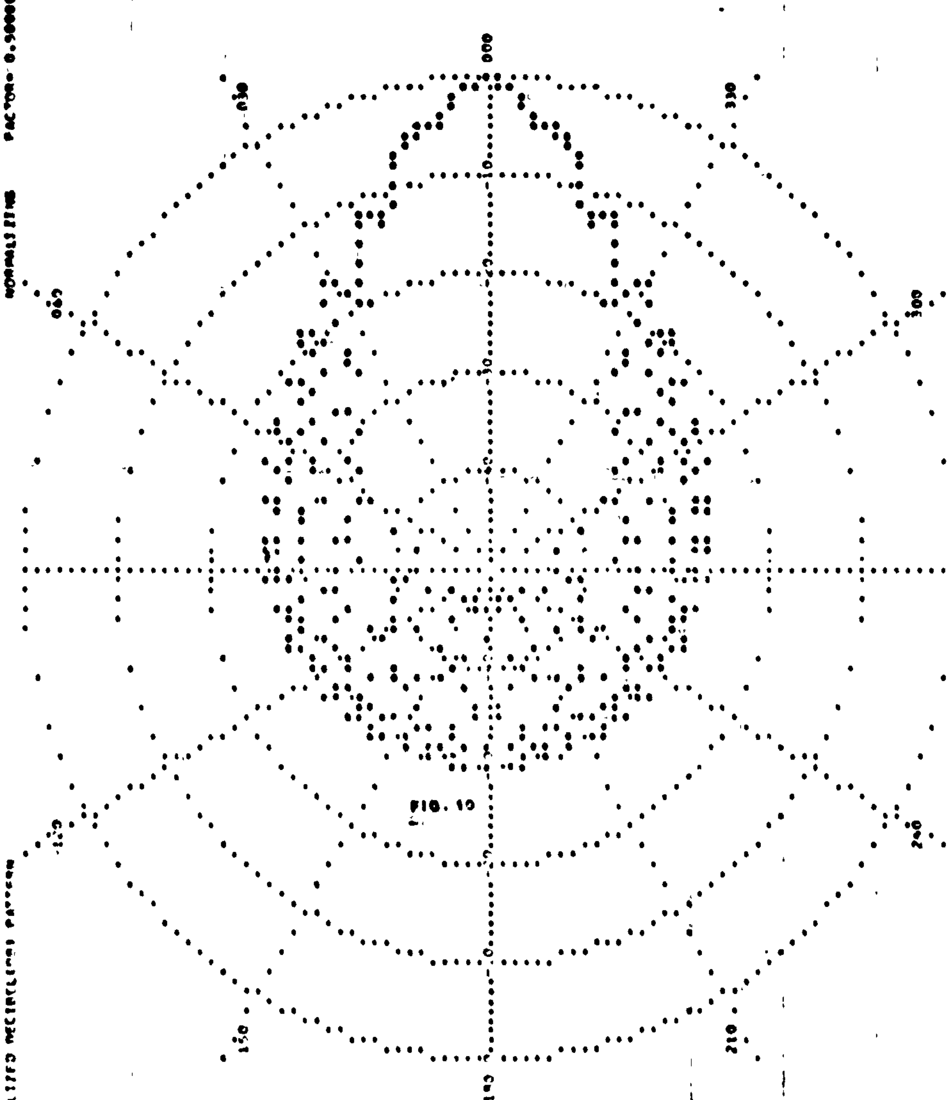


FIG. 10

THIS RUN IS FOR A TRANSDUCER OF THE SEGMENT TYPE.

NUMBER OF SUMMATIONS	20
TRANSDUCER LENGTH (METERS)	.20000E-01
TRANSDUCER WIDTH (RADIANS)	.18749E-01
RADIUS OF CYLINDER (METERS)	.20670E 00
FREQ (KHZ)	.85000E 02
TYPE OF PLOT (1) THETA VARYING, 2) PHI VARYING)	2
PLANE ANGLE HELD CONSTANT FOR PLOT	.90000E 02

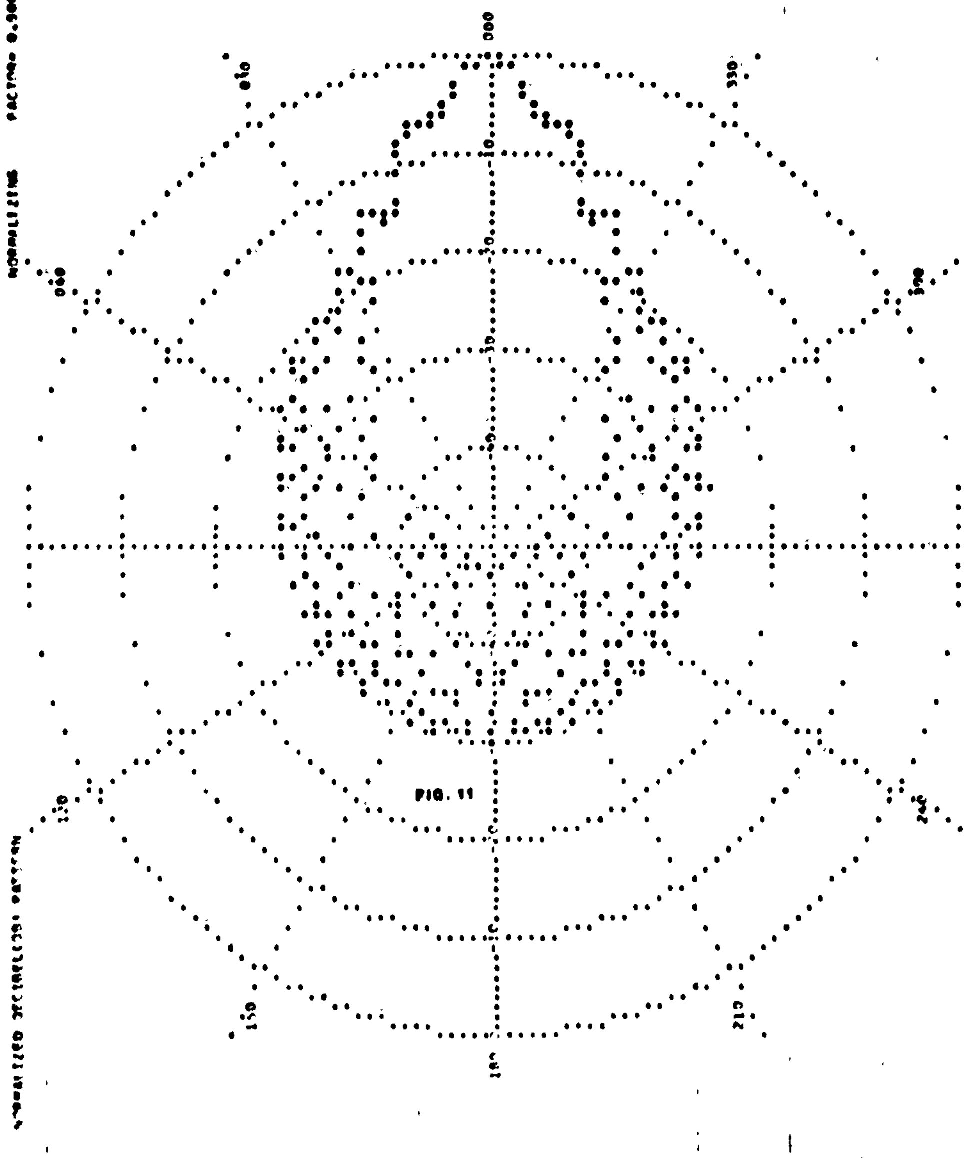


FIG. 11

THIS RUN IS FOR A TRANSDUCER OF THE SEGMENT TYPE.

NUMBER OF SUMMATIONS	29
TRANSDUCER [illegible] (METERS)	.20300E-01
[illegible] (RADIANS)	.18794E-01
RADIUS OF [illegible] (METERS)	.26670E 00
FREQUENCY(KHZ)	.25000E 02
TYPE OF PLOT (1: THETA VARYING, 2: PHI VARYING)	2
PLANE ANGLE HELD CONSTANT FOR PLOT	.90000E 02

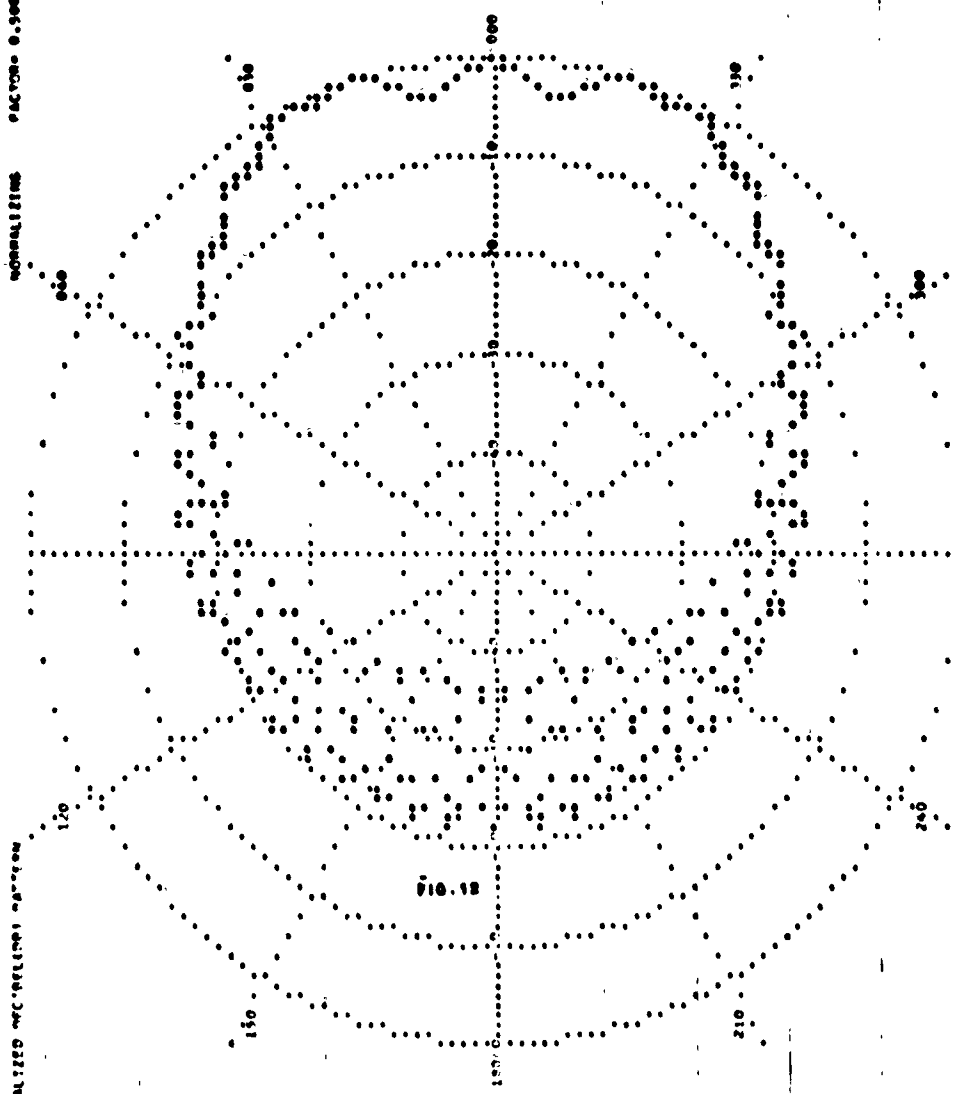


FIG.18

V. CONCLUSIONS AND RECOMMENDATIONS

Computer models which permit the computation of the sound radiation pattern for three (3) different source configurations mounted on a rigid cylindrical baffle have been developed.

The computer solutions agree favorably with previous mathematical results obtained by earlier investigators. Although empirical data are not available for confirmation of the patterns, it is considered that good agreement would result due to the manner in which the curvature of both the source and baffle were treated.

A. RECOMMENDATIONS FOR FUTURE DEVELOPMENT

Although the program as it exists is a useful design tool, several future modifications would enhance its capabilities. These items include: incorporation of amplitude and phase shading in the axial plane for the "Segment" design and incorporation of a 3-dimensional plotting package for all designs. Another improvement envisaged would be the linkage of this program with available parameter optimization programs [Ref. 6 and 7] using response surface methodology to determine specific input parameters for a desired pattern.

APPENDIX A

```
      DATA ASEG,APAT,ADISK/'SEGE','PATC','DISK'/
    1 READ(5,5,END=4) A
      WRITE(6,6)
      IF(A.EQ.ASEG) WRITE(6,8)
      IF(A.EQ.ASEG) CALL SEG
      IF(A.EQ.APAT) WRITE(6,9)
      IF(A.EQ.APAT) CALL PATCH
      IF(A.EQ.ADISK) WRITE(6,10)
      IF(A.EQ.ADISK) CALL DISK
      IF((A.EQ.ASEG).OR.(A.EQ.APAT).OR.(A.EQ.ADISK)) GO TO 1
    3 WRITE(6,7)
    4 STOP
C
    5 FORMAT(1A4)
    6 FORMAT(1H1)
    7 FORMAT(10X,'***FIRST DATA CARD MUST BE TYPE OF TRANSDUCER ***')
    8 FORMAT(40X,'THIS RUN IS FOR A TRANSDUCER OF THE SEGMENT TYPE.'///)
    9 FORMAT(40X,'THIS RUN IS FOR A TRANSDUCER OF THE PATCH TYPE.'////)
   10 FORMAT(40X,'THIS RUN IS FOR A TRANSDUCER OF THE DISK TYPE.'///)
      END
C     ..................................................................
C
C        SUBROUTINE BESJ
C
C        PURPOSE
C           COMPUTE THE J BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER
C
C        USAGE
C           CALL BESJ(X,N,BJ,D,IER)
C
C        DESCRIPTION OF PARAMETERS
C           X  -THE ARGUMENT OF THE J BESSEL FUNCTION DESIRED
C           N  -THE ORDER OF THE J BESSEL FUNCTION DESIRED
C           BJ -THE RESULTANT J BESSEL FUNCTION
C           D  -REQUIRED ACCURACY
C           IER-RESULTANT ERROR CODE WHERE
C              IER=0  NO ERROR
C              IER=1  N IS NEGATIVE
C              IER=2  X IS NEGATIVE OR ZERO
C              IER=3  REQUIRED ACCURACY NOT OBTAINED
C              IER=4  RANGE OF N COMPARED TO X NOT CORRECT (SEE REMARKS)
C
C        REMARKS
C           N MUST BE GREATER THAN OR EQUAL TO ZERO, BUT IT MUST BE
C           LESS THAN
C              20+10*X-X** 2/3   FOR X LESS THAN OR EQUAL TO 15
C              90+X/2            FOR X GREATER THAN 15
C
```

```
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           RECURRENCE RELATION TECHNIQUE DESCRIBED BY H. GOLDSTEIN AND
C           R.M. THALER,'RECURRENCE TECHNIQUES FOR THE CALCULATION OF
C           BESSEL FUNCTIONS',M.T.A.C.,V.13,PP.102-108 AND I.A. STEGUN
C           AND M. ABRAMOWITZ,'GENERATION OF BESSEL FUNCTIONS ON HIGH
C           SPEED COMPUTERS',M.T.A.C.,V.11,1957,PP.255-257
C
C     ..................................................................
C
      SUBROUTINE BESJ (X,N,BJ,D,IER)
C
      BJ = .0
      IF (N) 1,2,2
    1 IER = 1
      RETURN
    2 IF (X) 3,3,4
    3 IER = 2
      RETURN
    4 IF (X-15.) 5,5,6
    5 NTEST = 20.+10.*X-X**2/3
      GO TO 7
    6 NTEST = 90.+X/2.
    7 IF (N-NTEST) 9,8,8
    8 IER = 4
      RETURN
    9 IER = 0
      N1 = N+1
      BPREV = .0
C
C     COMPUTE STARTING VALUE OF M
C
      IF (X-5.) 10,11,11
   10 MA = X+6.
      GO TO 12
   11 MA = 1.4*X+60./X
   12 MB = N+IFIX(X)/4+2
      MZERO = MAX0(MA,MB)
C
C     SET UPPER LIMIT OF M
C
      MMAX = NTEST
C
      DO 21 M=MZERO,MMAX,3
C
C     SET F(M),F(M-1)
```

```
C
      FM1 = 1.0E-23
      FM = .0
      ALPHA = .0
      IF (M-(M/2)*2) 14,13,14
   13 JT = -1
      GO TO 15
   14 JT = 1
   15 M2 = M-2
C
      DO 18 K=1,M2
      MK = M-K
      BMK = 2.*FLOAT(MK)*FM1/X-FM
      FM = FM1
      FM1 = BMK
      IF (MK-N-1) 17,16,17
   16 BJ = BMK
   17 JT = -JT
      S = 1+JT
   18 ALPHA = ALPHA+BMK*S
C
      BMK = 2.*FM1/X-FM
      IF (N) 20,19,20
   19 BJ = BMK
   20 ALPHA = ALPHA+BMK
      BJ = BJ/ALPHA
      IF (ABS(BJ-BPREV)-ABS(D*BJ)) 22,22,21
   21 BPREV = BJ
C
      IER = 3
   22 RETURN
      END
C     ..................................................................
C
C        SUBROUTINE BESY
C
C        PURPOSE
C           COMPUTE THE Y BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER
C
C        USAGE
C           CALL BESY(X,N,BY,IER)
C
C        DESCRIPTION OF PARAMETERS
C           X  -THE ARGUMENT OF THE Y BESSEL FUNCTION DESIRED
C           N  -THE ORDER OF THE Y BESSEL FUNCTION DESIRED
C           BY -THE RESULTANT Y BESSEL FUNCTION
C           IER-RESULTANT ERROR CODE WHERE
C              IER=0  NO ERROR
```

```
C              IER=1   N IS NEGATIVE
C              IER=2   X IS NEGATIVE OR ZERO
C              IER=3   BY HAS EXCEEDED MAGNITUDE OF 10**70
C
C        REMARKS
C           VERY SMALL VALUES OF X MAY CAUSE THE RANGE OF THE LIBRARY
C           FUNCTION ALOG TO BE EXCEEDED
C           X MUST BE GREATER THAN ZERO
C           N MUST BE GREATER THAN OR EQUAL TO ZERO
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE
C           AS DESCRIBED BY A.J.M.HITCHCOCK,'POLYNOMIAL APPROXIMATIONS
C           TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED
C           FUNCTIONS', M.T.A.C., V.11,1957,PP.86-88, AND G.N. WATSON,
C           'A TREATISE ON THE THEORY OF BESSEL FUNCTIONS', CAMBRIDGE
C           UNIVERSITY PRESS, 1958, P. 62
C
C     ..................................................................
C
      SUBROUTINE BESY (X,N,BY,IER)
C
C     CHECK FOR ERRORS IN N AND X
C
      IF (N) 20,1,1
    1 IER = 0
      IF (X) 21,21,2
C
C     BRANCH IF X LESS THAN OR EQUAL 4
C
    2 IF (X-4.0) 4,4,3
C
C       COMPUTE Y0 AND Y1 FOR X GREATER THAN 4
C
    3 T1 = 4.0/X
      T2 = T1*T1
      P0 = ((((-.0000037043*T2+.0000173565)*T2-.0000487613)*T2+.00017343
     1)*T2-.001753062)*T2+.3989423
      Q0 = ((((.0000032312*T2-.0000142078)*T2+.0000342468)*T2-.000086979
     1)*T2+.0004564324)*T2-.01246694
      P1 = ((((.0000042414*T2-.0000200920)*T2+.0000580759)*T2-.000223203
     1)*T2+.002921826)*T2+.3989423
      Q1 = ((((-.0000036594*T2+.00001622)*T2-.0000398708)*T2+.0001064741
     1)*T2-.0006390400)*T2+.03740084
      A = 2.0/SQRT(X)
```

```
      B = A*T1
      C = X-.7853982
      Y0 = A*P0*SIN(C)+B*Q0*COS(C)
      Y1 = -A*P1*COS(C)+B*Q1*SIN(C)
      GO TO 9
C
C       COMPUTE Y0 AND Y1 FOR X LESS THAN OR EQUAL TO 4
C
    4 XX = X/2.
      X2 = XX*XX
      T = ALOG(XX)+.5772157
      SUM = 0.
      TERM = T
      Y0 = T
C
      DO 7 L=1,15
      IF (L-1) 5,6,5
    5 SUM = SUM+1./FLOAT(L-1)
    6 FL = L
      TS = T-SUM
      TERM = (TERM*(-X2)/FL**2)*(1.-1./(FL*TS))
    7 Y0 = Y0+TERM
C
      TERM = XX*(T-.5)
      SUM = 0.
      Y1 = TERM
C
      DO 8 L=2,16
      SUM = SUM+1./FLOAT(L-1)
      FL = L
      FL1 = FL-1.
      TS = T-SUM
      TERM = (TERM*(-X2)/(FL1*FL))*((TS-.5/FL)/(TS+.5/FL1))
    8 Y1 = Y1+TERM
C
      PI2 = .6366198
      Y0 = PI2*Y0
      Y1 = -PI2/X+PI2*Y1
C
C     CHECK IF ONLY Y0 OR Y1 IS DESIRED
C
    9 IF (N-1) 10,10,13
C
C     RETURN EITHER Y0 OR Y1 AS REQUIRED
C
   10 IF (N) 11,12,11
   11 BY = Y1
      GO TO 19
```

```
   12 BY = YO
      GO TO 19
C
C       PERFORM RECURRENCE OPERATIONS TO FIND YN(X)
C
   13 YA = YO
      YB = Y1
      K = 1
   14 T = FLOAT(2*K)/X
      YC = T*YB-YA
      IF (ABS(YC)-1.0E70) 16,16,15
   15 IER = 3
      RETURN
   16 K = K+1
      IF (K-N) 17,18,17
   17 YA = YB
      YB = YC
      GO TO 14
   18 BY = YC
   19 RETURN
   20 IER = 1
      RETURN
   21 IER = 2
      RETURN
      END
      SUBROUTINE DISK
C     READ INPUT DATA
C     THE INPUT DATA IS READ FROM TWO DATA CARDS.THE FIRST CARD CONTAINS
C     THE SUMMING LIMIT(N), OUTER RADIUS(ZZ), IN METERS,INNER RADIUS(ALP
C     IN METERS,RADIUS OF CYLINDRICAL BAFFLE(A) IN METERS,FREQUENCY
C     (FKHZ) IN KILOHERTZ. THE SECOND CARD CONTAINS PLOT INFORMATION.
C     FIRST ENTRY IS 'NPL'. IF NPL=1,A PLOT FOR THETA VARYING AS PHI
C     IS HELD CONSTANT, NPL=2, PHI VARIES AND THETA IS CONSTANT. BOTH
C     DATA CARDS USE SAME FORMAT.COLUMNS 1-2  ARE I2. REMAINING COLUMNS
C     11-20, 21-30, 31-40, 41-50  ARE  F10.7. THE SECOND ENTRY  ON THE
C     SECOND CARD IS THE VALUE (IN DEGREES) OF THE ANGLE HELD CONSTANT
      COMPLEX ASUMI
      COMPLEX GSUM
      COMPLEX RXD(500)
      COMPLEX B,SUM,HANK,RAD(500)
      DIMENSION RADMAG(500), DATAX(500), RMAGDB(500)
      DIMENSION BMAGDB(500)
C
C       INITIAL VALUES
C
      KFLAG = 0
      R = 100.
      PI = 3.1415927
```

```
      RHO = 1.54E6
      FACTOR = 1.
C
C
      READ (5,17) N,ZZ,ALPHA,A,FKHZ
      READ (5,18) NPL,ANGLE
C     CALCULATE INITIAL VALUES
      FREQ = FKHZ*1000.
      WRITE (6,19) N,ZZ,ALPHA,A,FKHZ,NPL,ANGLE
      ALAM = 1500./FREQ
      CK = 2.*PI/ALAM
      Z = ALPHA
      AK = CK*A
    1 IF (NPL.EQ.1) GO TO 2
      NPHI = 360
      PHI = 0
      NTH = 1
      GO TO 3
    2 NPHI = 1
      NTH = 360
      TH = 0
C
    3 DO 13 J=1,NPHI
      IF (NPL.EQ.1) PHI = ANGLE-1
      PHI = PHI+1
C
      DO 13 K=1,NTH
      L = J
      IF (NPL.EQ.1) L = K
      IF (NPL.EQ.2) TH = ANGLE-1
      IF (K.GT.180) TH = TH-1
      IF (K.LE.180) TH = TH+1
      IF (K.EQ.181) PHI = PHI+180
      APH = PHI/57.2957795
      ATH = TH/57.2957795
      ACTH = ABS(COS(ATH))
      ASTH = ABS(SIN(ATH))
      IF (ASTH.LE.0.0174525) GO TO 10
      GSUM = (0.,0.)
      SUM = (0.,0.)
C
      DO 9 I=1,N
      AI = I-1
      IF (AI.LT.1.) GO TO 4
      B = CMPLX(COS(AI*PI/2),-SIN(AI*PI/2))
      ASUMI = HANK(I,AK*SIN(ATH))
      IF (CABS(ASUMI).GT.(1.E+20)) GO TO 5
      SUM = (B*COS(AI*APH))/ASUMI
```

```
      GO TO 6
    4 SUM = 1.0/HANK(0,AK*SIN(ATH))
      GO TO 6
    5 SUM = (0.,0.)
    6 ZK = CK*COS(ATH)
      GTERM = 0.
      TERM = 0.
C
      DO 8 M=1,N,2
      AL = AI
      ARG = SQRT(ZK**2+(AL**2)/(A**2))
      TERMI = 0.
      IF (M.EQ.1) TERMI = .5
      IF ((ABS(TH-90.).LT.1.).AND.(I.EQ.1)) GO TO 7
      AR = Z*ARG
      CALL BESJ (AR,M,BJ,.1,IER)
      TERMI = BJ/AR
    7 TERM = TERM*TERMI
    8 GTERM = GTERM+TERM
C
      GSUM = GSUM+GTERM*SUM
    9 CONTINUE
C
      GKZ = (8.0*Z*Z*GSUM)/A
      RAD(L) = (2*RHO*GKZ)/(R*SIN(ATH))
      GO TO 11
   10 IF (L.EQ.1) RAD(L) = (0.,0.)
      IF (L.NE.1) RAD(L) = RAD(L-1)
      GO TO 12
   11 IF (KFLAG.EQ.1) RAD(L)=RAD(L)-2.*RXD(L)
   12 IF (KFLAG.EQ.0) RXD(L)=RAD(L)
   13 CONTINUE
C
      IF (KFLAG.EQ.0) RXD(1)=RXD(2)
      IF (KFLAG.EQ.1) RAD(1)=RAD(2)
      IF (KFLAG.EQ.1) GO TO 14
      KFLAG = 1
      Z = ZZ
      GO TO 1
C
   14 DO 15 I=1,360
      RMAGDB(I) = 0.
      RADMAG(I) = CABS(RAD(I))
      IF (RADMAG(I).LT.1.) GO TO 15
      RMAGDB(I) = 20.*ALOG10(RADMAG(I))
   15 IF (FACTOR.LT.RMAGDB(I)) FACTOR=RMAGDB(I)
C
C
```

```
      DO 16 I=1,360
      BMAGDB(I) = RMAGDB(I)+50.-FACTOR
   16 IF (BMAGDB(I).LT.0.) BMAGDB(I)=0.
C
      WRITE (6,20) (I,RAD(I),RADMAG(I),RMAGDB(I),BMAGDB(I),I=1,360)
      CALL POLPRT(1,RADMAG)
      CALL POLPRT(2,BMAGDB)
      RETURN
C
   17 FORMAT (I2,8X,4F10.7)
   18 FORMAT (L2,8X,F10.7)
   19 FORMAT(T30,'NUMBER OF SUMMATIONS',T80,I5/T30,'OUTER RADIUS OF DISK
     1(METERS)',T80,E10.5/T30,'INNER RADIUS OF DISK(METERS)',T80,E10.5/
     2T30,'RADIUS OF CYLINDER(METERS)',T80,E10.5/T30,'FREQUENCY(KHZ)',
     3T80,E10.5/T30,'TYPE OF PLOT (1: THETA VARYING, 2: PHI VARYING)'
     4,T80,I5/T30,'PLANE ANGLE HELD CONSTANT FOR PLOT',T80,E10.5////)
   20 FORMAT (1X,'ANGLE',19X,'RADIATION',21X,'MAGNITUDE',13X,
     1'MAGN(DB)',16X,'MAGN(DB)'/16X,'REAL',15X,'IMAG',56X,
     2'NORMALIZED'/(I5,5E20.5))
      END
C     SUBPROGRAM TO CALCULATE  HANKEL DERIVATIVE
      FUNCTION HANK (M,R)
      COMPLEX HANK,X,Y
      IF (R.LT.0.) R = -R
      CALL BESJ (R,M,BJ,.1,IER)
      CALL BESY (R,M,BY,IER)
      X = CMPLX(BJ,BY)
      CALL BESJ (R,M+1,BJ,.1,IER)
      CALL BESY (R,M+1,BY,IER)
      Y = CMPLX(BJ,BY)
      HANK = -Y+M*X/R
      RETURN
      END
      SUBROUTINE LINECK (X,Y)
C
C     THIS SUBROUTINE INSURES ALL GRID CHARACTORS LIE ON THE POLAR GRID
C
      COMMON ISYM,LINE
      COMMON /ITYPE/ IPRINT
      DATA MINUS/1H-/
      INTEGER Y
      DIMENSION ISYM(14), LINE(130)
      IF (Y.EQ.0) GO TO 3
      K = 0
      IF (X.LT.10.0) GO TO 6
C
C     SET UP AREAS OF "PERIOD" POLAR GRID POINT CHARACTERS
C
```

```
      I = INT(X)
      I = IABS(I)
      Z = ABS(X)
      IF ((Z-I).GT.0.5) I=I+1
    1 IF (Z.LT.10.0.OR.Z.GT.111.0) GO TO 2
      LINE(I) = ISYM(2)
      LINE(60) = ISYM(3)
      LINE(62) = ISYM(3)
      K = K+1
      IF (K.EQ.2) GO TO 2
      I = 122-I
      GO TO 1
    2 LINE(61) = ISYM(2)
      IF (Y.NE.0) GO TO 6
C
    3 DO 4 K=11,111
      LINE(K) = ISYM(2)
    4 CONTINUE
C
C
C     FILL IN GRID NUMBER LABELS ON HORIZONTAL AXIS
C
      IF (IPRINT.GT.1) GO TO 5
      LINE(11) = ISYM(7)
      LINE(20) = ISYM(10)
      LINE(21) = ISYM(5)
      LINE(22) = ISYM(11)
      LINE(30) = ISYM(9)
      LINE(31) = ISYM(5)
      LINE(32) = ISYM(11)
      LINE(40) = ISYM(8)
      LINE(41) = ISYM(5)
      LINE(42) = ISYM(11)
      LINE(50) = ISYM(7)
      LINE(51) = ISYM(5)
      LINE(52) = ISYM(11)
      LINE(61) = ISYM(1)
      LINE(70) = ISYM(7)
      LINE(71) = ISYM(5)
      LINE(72) = ISYM(11)
      LINE(80) = ISYM(8)
      LINE(81) = ISYM(5)
      LINE(82) = ISYM(11)
      LINE(90) = ISYM(9)
      LINE(91) = ISYM(5)
      LINE(92) = ISYM(11)
      LINE(100) = ISYM(10)
      LINE(101) = ISYM(5)
```

```
      LINE(102) = ISYM(11)
      LINE(111) = ISYM(7)
      GO TO 6
    5 LINE(11) = ISYM(6)
      LINE(20) = MINUS
      LINE(21) = ISYM(7)
      LINE(22) = ISYM(6)
      LINE([illegible]) = MINUS
      LINE(31) = ISYM(8)
      LINE(32) = ISYM(6)
      LINE([illegible]) = MINUS
      LINE(41) = ISYM(3)
      LINE([illegible]) = ISYM(6)
      LINE([illegible]) = [illegible]
      LINE([illegible]) = ISYM(10)
      LINE([illegible]) = ISYM(6)
      LINE(61) = ISYM(1)
      LINE([illegible]) = [illegible]
      LINE(71) = ISYM(10)
      LINE(72) = ISYM(6)
      LINE([illegible]) = [illegible]
      LINE([illegible]) = ISYM(9)
      LINE([illegible]) = ISYM(6)
      LINE([illegible]) = MINUS
      LINE([illegible]) = ISYM(8)
      LINE([illegible]) = ISYM(6)
      LINE([illegible]) = MINUS
      LINE([illegible]) = ISYM(7)
      LINE([illegible]) = ISYM(6)
      LINE([illegible]) = ISYM(6)
    6 CONTINUE
      RETURN
      END
      SUBROUTINE NUMB (Y)
C
C     THIS SUBROUTINE PUTS DEGREE NUMBERS ON POLAR GRID
C
      COMMON ISYM,LINE
      INTEGER Y
      DIMENSION ISYM(14), LINE(130)
      IF (Y.NE.37) GO TO 1
      LINE(33) = ISYM(7)
      LINE(34) = ISYM(8)
      LINE(35) = ISYM(6)
      LINE(37) = ISYM(6)
      LINE([illegible]) = ISYM(12)
      LINE([illegible]) = ISYM(6)
    1 IF (Y.NE.21) GO TO 2
```

```
      LINE(12) = ISYM(7)
      LINE(13) = ISYM(11)
      LINE(14) = ISYM(6)
      LINE(108) = ISYM(6)
      LINE(109) = ISYM(9)
      LINE(110) = ISYM(6)
    2 IF (Y.NE.0) GO TO 3
      LINE(7) = ISYM(7)
      LINE(8) = ISYM(13)
      LINE(9) = ISYM(6)
      LINE(113) = ISYM(6)
      LINE(114) = ISYM(5)
      LINE(115) = ISYM(6)
    3 IF (Y.NE.-21) GO TO 4
      LINE(12) = ISYM(3)
      LINE(13) = ISYM(7)
      LINE(14) = ISYM(6)
      LINE(108) = ISYM(9)
      LINE(109) = ISYM(9)
      LINE(110) = ISYM(6)
    4 IF (Y.NE.-37) GO TO 5
      LINE(33) = ISYM(3)
      LINE(34) = ISYM(10)
      LINE(35) = ISYM(6)
      LINE(37) = ISYM(9)
      LINE(38) = ISYM(6)
      LINE(39) = ISYM(6)
    5 CONTINUE
      RETURN
      END
      SUBROUTINE PATCH
C     READ INPUT DATA
C     THE INPUT DATA IS READ FROM TWO DATA CARDS.THE FIRST CARD CONTAINS
C     THE SUMMING LIMIT (N),TRANSDUCER HEIGHT(ZZ) IN METERS,WIDTH(ALPHA)
C     IN RADIANS,RADIUS OF CYLINDRICAL BAFFLE (A) IN METERS,FREQUENCY
C     (FKHZ) IN KILOHERTZ. THE SECOND CARD CONTAINS PLOT INFORMATION.
C     FIRST ENTRY IS 'NPL'. IF NPL=1,A PLOT FOR THETA VARYING AS PHI
C     IS HELD CONSTANT, NPL=2, PHI VARIES AND THETA IS CONSTANT. BOTH
C     DATA CARDS USE SAME FORMAT.COLUMNS 1-2  ARE I2. REMAINING COLUMNS
C     11-20, 21-30, 31-40, 41-50  ARE  F10.7. THE SECOND ENTRY  ON THE
C     SECOND CARD IS THE VALUE (IN DEGREES) OF THE ANGLE HELD CONSTANT
      COMPLEX ASUM
      COMPLEX RXO(500)
      COMPLEX SUMT
      COMPLEX B,SUM,HANK,RAD(500)
      DIMENSION RADMAG(500), DATAX(500), RMAGDB(500)
      DIMENSION RMAGDB(500)
C
```

```
C       INITIAL VALUES
C
        KFLAG = 0
        R = 100.
        PI = 3.1415927
        RHO = 1.54E6
        FACTOR = 1.
C
C
        READ (5,13) N,ZZ,ALPHA,A,FKHZ
        READ (5,13) NPL,ANGLE
C       CALCULATE INITIAL VALUES
        FREQ = FKHZ*1000.
        WRITE (6,14) N,ZZ,ALPHA,A,FKHZ,NPL,ANGLE
        ALAM = 1500./FREQ
        CK = 2.*PI/ALAM
        Z = ZZ/2.
        AK = CK*A
      1 IF (NPL.EQ.1) GO TO 2
        NPHI = 360
        PHI = 0
        NTH = 1
        GO TO 3
      2 NPHI = 1
        NTH = 360
        TH = 0
C
      3 DO 9 J=1,NPHI
        IF (NPL.EQ.1) PHI = ANGLE-1
        PHI = PHI+1
C
        DO 9 K=1,NTH
        L = J
        IF (NPL.EQ.1) L = K
        IF (NPL.EQ.2) TH = ANGLE-1
        IF (K.GT.180) TH = TH-1
        IF (K.LE.180) TH = TH+1
        IF (K.EQ.181) PHI = PHI+180
        APH = PHI/57.2957795
        ATH = TH/57.2957795
        ACTH = ABS(COS(ATH))
        ASTH = ABS(SIN(ATH))
        IF (ASTH.LE.0.0174525) GO TO 6
        SUM = (ALPHA/PI)/HANK(0,AK*SIN(ATH))
C
        DO 4 I=1,N
        AI = I
        B = CMPLX(COS(AI*PI/2),-SIN(AI*PI/2))
```

```
      ASUMI = HANK(I,AK*SIN(ATH))
      IF (CABS(ASUMI).GT.(1.E+20)) GO TO 4
      SUM = SUM+(2*SIN(AI*ALPHA)*B*COS(AI*APH))/(AI*PI*ASUMI)
    4 CONTINUE
C
      IF (ACTH.LT.0.0174524) GO TO 5
      RAD(L) = 2*RHO*SIN(CK*Z*COS(ATH))/(CK*COS(ATH)*SIN(ATH))*SUM/(R*PI
     1)
    5 RAD(L) = 2*RHO*Z*SUM/(R*PI)
      GO TO 7
    6 IF (L.EQ.1) RAD(L) = (0.,0.)
      IF (L.NE.1) RAD(L) = RAD(L-1)
      GO TO 8
    7 IF (KFLAG.EQ.1) RAD(L)=RAD(L)-2.*RXD(L)
    8 IF (KFLAG.EQ.0) RXD(L)=RAD(L)
    9 CONTINUE
C
      IF (KFLAG.EQ.0) RXD(1)=RXD(2)
      IF (KFLAG.EQ.1) RAD(1)=RAD(2)
      IF (KFLAG.EQ.1) GO TO 10
      KFLAG = 1
      Z = 3.*Z
      ALPHA = 3.*ALPHA
      GO TO 1
C
   10 DO 11 I=1,360
      RMAGDB(I) = 0.
      RADMAG(I) = CABS(RAD(I))
      IF (RADMAG(I).LT.1.) GO TO 11
      RMAGDB(I) = 20.*ALOG10(RADMAG(I))
   11 IF (FACTOR.LT.RMAGDB(I)) FACTOR=RMAGDB(I)
C
C
      DO 12 I=1,360
      BMAGDB(I) = RMAGDB(I)+50.-FACTOR
   12 IF (BMAGDB(I).LT.0.) BMAGDB(I)=0.
C
      WRITE (6,15) (I,RAD(I),RADMAG(I),RMAGDB(I),BMAGDB(I),I=1,360)
      CALL POLPRT (1,RADMAG)
      CALL POLPRT (2,BMAGDB)
      RETURN
C
   13 FORMAT (I2,8X,4F10.7)
   14 FORMAT(T30,'NUMBER OF SUMMATIONS',T80,I5/T30,'TRANSDUCER HEIGHT(
     1METERS)',T80,E10.5/T30,'TRANSDUCER WIDTH(RADIANS)',T80,E10.5/
     2T30,'RADIUS OF CYLINDER(METERS)',T80,E10.5/T30,'FREQUENCY(KHZ)',
     3T80,E10.5/T30,'TYPE OF PLOT (1: THETA VARYING, 2: PHI VARYING)'
     4,T80,I5/T30,'PLANE ANGLE HELD CONSTANT FOR PLOT',T80,E10.5////)
```

```
   15 FORMAT (1X,'ANGLE',19X,'RADIATION',21X,'MAGNITUDE',13X,
     1'MAGN(DB)',16X,'MAGN(DB)'/16X,'REAL',15X,'IMAG',56X,
     2'NORMALIZED'/(I5,5E20.5))
      END
      SUBROUTINE PIPLOT (IYY,S)
C
C     THIS SUBROUTINE SETS UP POLAR GRID INFORMATION
C
      COMMON ISYM,LINE
      DIMENSION LINE(130), ISYM(14), ISYN(14)
      DATA ISYN/1H+,1H.,1H ,1H*,1H/,1H0,1H1,1H2,1H3,1H4,1H5,1H6,1H8,1H9/
      INTEGER Y,YY,W
C
C     SET UP ISYM FROM ISYN FOR COMMON
C
C
      DO 1 K=1,14
      ISYM(K) = ISYN(K)
    1 CONTINUE
C
C
C     CLEAR LINE AND SET TO BLANK
C
C
      DO 2 I=1,130
    2 LINE(I) = ISYM(3)
C
      Y = 41-IYY
      IF (Y.EQ.0) GO TO 7
C
C     SET UP EQUATIONS FOR CONCENTRIC CIRCLES
C
      YY = Y*Y
      Z = (YY*2.5/2)*S
      X = 61.0+SQRT(2500.0-Z)
      CALL LINECK (X,Y)
      IF (Y.GT.32.OR.Y.LT.-32) GO TO 3
      X = 61.0+SQRT(1600.0-Z)
      CALL LINECK (X,Y)
    3 IF (Y.GT.24.OR.Y.LT.-24) GO TO 4
      X = 61.0+SQRT(900.0-Z)
      CALL LINECK (X,Y)
    4 IF (Y.GT.16.OR.Y.LT.-16) GO TO 5
      X = 61.0+SQRT(400.0-Z)
      CALL LINECK (X,Y)
    5 IF (Y.GT.8.OR.Y.LT.-8) GO TO 6
      X = 61.0+SQRT(100-Z)
      CALL LINECK (X,Y)
```

```
C     SET UP EQUATIONS FOR MULTIPLES OF 30 DEGREES
    6 X = 61.0+1.732051*Y*S
      CALL LINECK (X,Y)
      X = 61.0+Y*S/1.732051
    7 CALL LINECK (X,Y)
C
C     PUT IN POLAR PLOT NUMBER LABELS
C
      CALL NUMB (Y)
      W = IABS(Y)
C
C     FILL IN POLAR PLOT AT 000, 090, 180, AND 270
C
      IF (W.NE.40) GO TO 8
      LINE(55) = ISYM(2)
      LINE(57) = ISYM(2)
      LINE(59) = ISYM(2)
      LINE(63) = ISYM(2)
      LINE(65) = ISYM(2)
      LINE(67) = ISYM(2)
    8 IF (W.NE.32) GO TO 9
      LINE(56) = ISYM(2)
      LINE(58) = ISYM(2)
      LINE(60) = ISYM(2)
      LINE(62) = ISYM(2)
      LINE(64) = ISYM(2)
      LINE(66) = ISYM(2)
    9 IF (W.NE.24) GO TO 10
      LINE(57) = ISYM(2)
      LINE(59) = ISYM(2)
      LINE(60) = ISYM(2)
      LINE(62) = ISYM(2)
      LINE(63) = ISYM(2)
      LINE(65) = ISYM(2)
   10 IF (W.NE.16) GO TO 11
      LINE(58) = ISYM(2)
      LINE(60) = ISYM(2)
      LINE(62) = ISYM(2)
      LINE(64) = ISYM(2)
   11 IF (W.NE.08) GO TO 12
      LINE(59) = ISYM(2)
      LINE(63) = ISYM(2)
   12 CONTINUE
      RETURN
      END
      SUBROUTINE POLPRT (ITYPE,Y)
      COMMON ISYM,LINE
      DIMENSION NAME(80)
```

```
      DIMENSION X(360), Y(360), DATAX(360), DATAY(360), LINE(130), ISYM(
     114)
      COMMON /ITYPE/ IPRINT
      IPRINT = ITYPE
      N = 360
      DIM = 1.0
      NST = 1
      KST = 1
C
C     S IS SCALE FACTOR OF PRINTER:
C     ABSESSA CHAR. PER INCH / ORDINATE CHAR. PER INCH
C
      S = 10.0/8.0
C
C     ZERO DATAX AND DATAY
C
C
C
      DO 1 IA=1,N
      D = IA
      DATAX(IA) = 0.0
      DATAY(IA) = 0.0
    1 X(IA) = D*3.1415927/180.0
C
C
C     FACTOR IS THE NORMALIZING DIVISOR
C
      FACTOR = Y(1)
C
C
      DO 2 IA=2,N
    2 IF (FACTOR.LT.Y(IA)) FACTOR=Y(IA)
C
C
C     NORMALIZE DATA TO ONE
C
C
C
      DO 3 IA=1,N
    3 Y(IA) = Y(IA)/FACTOR
C
C
      IF(IPRINT.EQ.1)WRITE(6,7) FACTOR
      IF(IPRINT.EQ.2)WRITE(6,8) FACTOR
C
C     FILL DATAX AND DATAY ARRAY FROM X AND Y ARRAY
```

```
C
C
C
      DO 10 IA=1,N
      DATA X(IA) = Y(IA)*COS(X(IA))
   10 DATA Y(IA) = Y(IA)*SIN(X(IA))
C
C
C     SORT DATA BY ORDINATE MAGNITUDE
C
      CALL SART (DATAX,DATAY,N)
C
C     DATAX AND DATAY ARE SORTED BY DESENDING MAGNITUDE ON THE DATAY VAL
C     SET UP FOR PLOTTING POLAR GRID WITH DATA
C
C
C
      DO 6 IYY=1,81
C
      CALL PTPLOT (IYY,S)
C
C     LINE IS RETURNED WITH POLAR GRID INFORMATION
C
C     SET UP 'Y' BIN SIZE UPPER AND LOWER LIMITS
C     ULL IS THE LOWER BIN LIMIT
C     UL IS THE UPPER BIN LIMIT
C
      BIN = DIM/80.0
      ULL = DIM-(2*(IYY-1)*BIN
      UL = ULL+2*BIN
C
C
C     CYCLE THROUGH DATA TO FIND WHICH ONES FALL IN 'Y' BINS
C
C
      IF (NST.GT.N) GO TO 5
C
C
      DO 4 JJ=NST,N
      IF (DATAY(JJ).LT.ULL) GO TO 5
      KST = JJ
      AMAG = SQRT(DATAX(JJ)*DATAX(JJ)+DATAY(JJ)*DATAY(JJ))
C
C     CHECK THAT MAGNITUDE IS NOT OVER DIM
C
      IF (AMAG.GT.DIM) GO TO 4
C
C     OK IS THE FINAL LINE POSITION FOR THE '*'
```

```
C
      OK = DATAX(JJ)*S*40.0/DIM+61.0
      IF (OK.LT.10.0) GO TO 4
      K = INT(OK)
      K = IABS(K)
      OK = ABS(OK)
      IF ((OK-K).GT.0.5) K=K+1
      IF (OK.LT.10.0.OR.OK.GT.111.0) GO TO 4
      LINE(K) = ISYM(4)
    4 CONTINUE
C
C
    5 CONTINUE
      NST = KST+1
C
C     PRINT OUT ONE LINE OF PLOT
C
      WRITE (6,9) LINE
    6 CONTINUE
C
C
      RETURN
C
C
    7 FORMAT(1H1,' NORMALIZED MAGNITUDE PATTERN',T90,'NORMALIZING FACTOR
     1=',E12.5)
    8 FORMAT(1H1,' NORMALIZED DECIBEL(DB) PATTERN',T90,'NORMALIZING
     1FACTOR=',E12.5)
    9 FORMAT (1X,130A1)
      END
      SUBROUTINE SART (DATAX,DATAY,N)
      DIMENSION DATAX(500), DATAY(500)
C
C     THIS ROUTINE SORTS DATA IN DATAY BY MAGNITUDE
C
      NN = N-1
C
      DO 2 I=1,NN
      NM = I+1
C
      DO 1 J=NM,N
      IF (DATAY(I).GE.DATAY(J)) GO TO 1
      STOR = DATAY(I)
      DATA Y(I) = DATAY(J)
      DATA Y(J) = STOR
      STOR = DATAX(I)
      DATA X(I) = DATAX(J)
      DATA X(J) = STOR
```

```
    1 CONTINUE
C
    2 CONTINUE
C
      RETURN
      END
      SUBROUTINE SEG
C     READ INPUT DATA
C     THE INPUT DATA IS READ FROM TWO DATA CARDS.THE FIRST CARD CONTAINS
C     THE SUMMING LIMIT (N),TRANSDUCER HEIGHT(ZZ) IN METERS,WIDTH(ALPHA)
C     IN RADIANS,RADIUS OF CYLINDRICAL BAFFLE (A) IN METERS,FREQUENCY
C     (FKHZ) IN KILOHERTZ. THE SECOND CARD CONTAINS PLOT INFORMATION.
C     FIRST ENTRY IS 'NPL'. IF NPL=1,A PLOT FOR THETA VARYING AS PHI
C     IS HELD CONSTANT, NPL=2, PHI VARIES AND THETA IS CONSTANT. BOTH
C     DATA CARDS USE SAME FORMAT.COLUMNS 1-2  ARE I2. REMAINING COLUMNS
C     11-20, 21-30, 31-40, 41-50  ARE  F10.7. THE SECOND ENTRY  ON THE
C     SECOND CARD IS THE VALUE (IN DEGREES) OF THE ANGLE HELD CONSTANT
C     TO OBTAIN AN AMPLITUDE AND PHASE SHADING PATTERN, ENTER
C     EITHER A '1' OR '2' IN COLUMN 2 OF THE THIRD DATA CARD.
C     IF NO SHADING PATTERN IS DESIRED, THE 3RD DATA CARD IS LEFT BLANK.
C     IF THE 3RD DATA CARD CONTAINS A '1' OR A '2',THE FOURTH(4) AND
C     SUCCEEDING DATA CARDS WILL CONTAIN THE AMPLITUDE AND PHASE
C     SHADING INFORMATION. TO ENTER THIS INFORMATION, THE FOURTH DATA
C     CARD MUST CONTAIN THE TOTAL NUMBER OF ELEMENTS TO BE SHADED
C     (MAXIUM OF 120 ELEMENTS) IN COLUMNS 1-3.  THAT IS, FOR FIVE
C     ELEMENTS A '5' IN COLUMN 3; FOR 24 ELEMENTS, '24' ENTERED IN
C     COLUMNS '2' AND '3'.  A SEPARATE DATA CARD FOR EACH OF THE
C     ELEMENTS, FOR THE TOTAL NUMBER OF ELEMENTS SPECIFIED ON DATA CARD
C     FOUR(4) IS THEN ENTERED CONSECUTIVELY IN THE FOLLOWING FORMAT:
C     COLUMNS1-3, SPECIFIES ELEMENT NUMBER, COLUMNS 11-20(IN F10.5
C     FORMAT) SPECIFIES THE AMPLITUDE SHADING, AND COLUMNS 21-30(IN
C     F10.5 FORMAT) SPECIFIES PHASE SHADING ENTERED AS AN ANGLE
C     (EXAMPLE, '-92.0' FOR -92.0 DEGREES).
      COMPLEX ASUMI
      COMPLEX SUMI
      DIMENSION ARPDB(500)
      COMPLEX TERM(50)
      COMPLEX CORP
      COMPLEX AASS
      DIMENSION AMP(120), SHADE(120), ASHAD(120), ARRAD(500)
      DIMENSION BMAGDB(500)
      COMPLEX RRAD(500)
      COMPLEX B,SUM,HANK,RAD(500)
      DIMENSION RADMAG(500), DATAX(500), RMAGDB(500)
C
C     INITIAL VALUES
C
      R = 100.
```

```
      PI = 3.1415927
      RHO = 1.54E6
      FACTOR = 1.
      BOSS = 1.
C
C
      READ (5,20) N,ZZ,ALPHA,A,FKHZ
      READ (5,20) NPL,ANGLE
C     CALCULATE INITIAL VALUES
      FREQ = FKHZ*1000.
      WRITE (6,21) N,ZZ,ALPHA,A,FKHZ,NPL,ANGLE
      ALAM = 1500./FREQ
      CK = 2.*PI/ALAM
      Z = ZZ/2.
      AK = CK*A
      IF (NPL.EQ.1) GO TO 1
      NPHI = 360
      PHI = 0
      NTH = 1
      GO TO 2
    1 NPHI = 1
      NTH = 360
      TH = 0
C
C
    2 DO 6 J=1,NPHI
      IF (NPL.EQ.1) PHI = ANGLE-1
      PHI = PHI+1
C
C
      DO 6 K=1,NTH
      L = J
      IF (NPL.EQ.1) L = K
      IF (NPL.EQ.2) TH = ANGLE-1
      IF (K.GT.180) TH = TH-1
      IF (K.LE.180) TH = TH+1
      IF (K.EQ.181) PHI = PHI+180
      APH = PHI/57.2957795
      ATH = TH/57.2957795
      ACTH = ABS(COS(ATH))
      ASTH = ABS(SIN(ATH))
      IF (ASTH.LE.0.0174525) GO TO 4
      SUM = (ALPHA/PI)/HANK(0,AK*SIN(ATH))
C
C
      DO 3 I=1,N
      AI = I
      B = CMPLX(COS(AI*PI/2),-SIN(AI*PI/2))
```

```
      ASUMI = HANK(I,AK*SIN(ATH))
      IF (CABS(ASUMI).GT.(1.E+20)) GO TO 3
      SUM = SUM+(2*SIN(AI*ALPHA)*B*COS(AI*APH))/(AI*PI*ASUMI)
    3 CONTINUE
C
C
      IF (ACTH.LT.0.0174524) GO TO 5
      RAD(L) = 2*RHO*SIN(CK*Z*COS(ATH))/(CK*COS(ATH)*SIN(ATH))*SUM/(R*PI
     1)
      GO TO 6
    4 IF (L.EQ.1) RAD(L) = (0.,0.)
      IF (L.NE.1) RAD(L) = RAD(L-1)
      GO TO 6
    5 RAD(L) = 2*RHO*Z*SUM/(R*PI)
    6 CONTINUE
C
C
      RAD(1) = RAD(2)
C
C
      DO 7 I=1,360
      RMAGDB(I) = 0.
      RADMAG(I) = CABS(RAD(I))
      IF (RADMAG(I).LT.1.) GO TO 7
      RMAGDB(I) = 20.*ALOG10(RADMAG(I))
    7 IF (FACTOR.LT.RMAGDB(I)) FACTOR=RMAGDB(I)
C
C
C
C
      DO 8 I=1,360
      BMAGDB(I) = RMAGDB(I)+50.-FACTOR
    8 IF (BMAGDB(I).LT.0.) BMAGDB(I)=0.
C
C
      WRITE (6,22) (I,RAD(I),RADMAG(I),RMAGDB(I),BMAGDB(I),I=1,360)
      CALL POLPRT (1,RADMAG)
      CALL POLPRT (2,BMAGDB)
      READ (5,20) NPL
      IF (NPL.EQ.0) RETURN
      READ (5,19) NUMB
C
      DO 9 I=1,120
      AMP(I) = 0.
    9 SHADE(I) = 0.
C
C
      DO 10 I=1,NUMB
```

```
      READ (5,19) L,X,Y
      AMP(L) = X
   10 SHADE(L) = Y
C
      WRITE (6,23)
      WRITE (6,24) (I,AMP(I),SHADE(I),I=1,120)
C
C
      DO 11 I=1,120
   11 ASHAD(I) = SHADE(I)/57.2957795
C
C
C
C
      DO 12 I=1,360
   12 RRAD(I) = (0.,0.)
C
C
C
C
      DO 14 I=1,N
      AI = I-1
      IF (AI.EQ.0.) GO TO 13
      B = CMPLX(COS(AI*PI/2),-SIN(AI*PI/2))
      TERM(I) = (2*SIN(AI*ALPHA)*B)/(AI*PI*HANK(I,AK))
      GO TO 14
   13 TERM(I) = (ALPHA/PI)/HANK(0,AK)
   14 CONTINUE
C
C
      CONS = (2*RHO*Z)/(K*PI)
      PHI = 0
C
C
      DO 16 M=1,360
C
C
      DO 16 K=1,120
      PHI = (K-1)*3+M
      APHI = PHI/57.2957795
      AASS = CMPLX(COS(ASHAD(K)),SIN(ASHAD(K)))
      SUM = (0.,0.)
C
C
      DO 15 J=1,N
      AJ = J-1
   15 SUM = SUM+(TERM(J))*(COS(AJ*APHI))
C
```

```
C
      RRAD(M) = AMP(K)*AASS*SUM*CONS+RRAD(M)
   16 CONTINUE
C
C
C
C
      DO 17 I=1,360
      ARRDB(I) = 0.
      ARRAD(I) = (CABS(RRAD(I)))
      IF (ARRAD(I).LT.1.) GO TO 17
      ARRDB(I) = 20*ALOG10(ARRAD(I))
   17 IF (BOSS.LT.ARRDB(I)) BOSS=ARRDB(I)
C
C
C
C
      DO 18 I=1,360
      ARRDB(I) = ARRDB(I)+50.-BOSS
   18 IF (ARRDB(I).LT.0.) ARRDB(I)=0.
C
C
      WRITE (6,25) (I,RRAD(I),ARRAD(I),ARRDB(I),I=1,360)
      CALL POLPRT (1,ARRAD)
      CALL POLPRT (2,ARRDB)
C
      RETURN
C
   19 FORMAT (I3,7X,2F10.5)
   20 FORMAT (I2,8X,4F10.7)
   21 FORMAT(T30,'NUMBER OF SUMMATIONS',T80,I5/T30,'TRANSDUCER HEIGHT(
     1METERS)',T80,E10.5/T30,'TRANSDUCER WIDTH(RADIANS)',T80,E10.5/
     2T30,'RADIUS OF CYLINDER(METERS)',T80,E10.5/T30,'FREQUENCY(KHZ)',
     3T80,E10.5/T30,'TYPE OF PLOT (1  THETA VARYING, 2: PHI VARYING)'
     4,T80,I5/T30,'PLANE ANGLE HELD CONSTANT FOR PLOT',T80,E10.5////)
   22 FORMAT (1X,'ANGLE',19X,'RADIATION',21X,'MAGNITUDE',13X,
     1'MAGN(DB)',16X,'MAGN(DB)'/15X,'REAL',15X,'IMAG',56X,
     2'NORMALIZED'/(I5,5E20.5))
   23 FORMAT (4(1X,'ELEMENT',2X,'AMPLITUDE',2X,'PHASE',4X))
   24 FORMAT (4X,I3,6X,F4.1,4X,F6.1,7X,I3,6X,F4.1,4X,F6.1,7X,I3,6X,F4.1,
     14X,F6.1,7X,I3,6X,F4.1,4X,F6.1,/)
   25 FORMAT (1X,'ANGLE',19X,'RADIATION',21X,'MAGNITUDE',13X,
     2'MAGN(DB)'/16X,'REAL',15X,'IMAG',44X,'NORMALIZED'/(I5,4E20.5))
      END
```

MAIN

Purpose: To control program

Method: The program inspects the first four (4) letters of the coded titles (DISK, PATCH, SEGEMENT - Note the spelling of the "Segment" design) to transfer control to the proper subroutine. It also writes out the first line of output identifying the design configuration being calculated.

Called By: First Input Data Card

Calls To: DISK
PATCH
SEG

HANK

Purpose: To calculate a Hankel derivative given the order and argument.

Method: This subroutine is a function subroutine which calculates the Hankel derivative by the recurrence relation, $H'_{(N)}(R) = \frac{N}{R} H_{(N)}(R) - H_{(N+1)}(R)$. The Hankel function of order (N) and order (N+1) are calculated by combining the "J" and "Y" Bessel functions as a complex number.

Called By: DISK
PATCH
SEGMENT

Calls To: BESY
BESJ

DISK

Purpose: To calculate the radiation pattern for the "Disk" design.

Method: The "DO" loop ending with statement number nine (9) calculates the pattern for the "DISK" design. It contains within it, a second "DO" loop ["DO loop ending with statement number eight (8)] which calculates the infinite series composed of the odd Bessel Functions. The entire loop is executed two times through the use of the indicator, "KFLAG." After the second execution of the loop, twice the results of the first execution are subtracted from those of the second execution. The statement, "IF(KFLAG.EQ.1) GO TO 14" transfers the results of the calculations to the plotting package.

Called By: MAIN

Calls To: HANK
POLPLT

PATCH

Purpose: To calculate the radiation pattern for the "Patch" design.

Method: The basic equations derived for the "Patch" design are coded in the "DO" loop ending with statement number nine (9). The pattern is achieved by executing that "DO" loop twice. In the first execution of the loop, the pattern for the oppositely phase shaded center element is calculated. Subsequently, "KFLAG" (an indicator of how many times the

loop has been executed) is updated, and the dimensions of the element changed to those of the outer (larger) element. After the second execution of the loop, two (2) times the first pattern is subtracted from the pattern obtained during the second execution of the loop. The "IF" statement, IF(KFLAG.EQ.1) GO TO 10" transfers the calculations to the plotting subroutines.

Called By: MAIN

Calls To: HANK
POLPRT

SEG

Purpose: To calculate the radiation pattern for the "Segment" design.

Method: This subroutine can be thought to consist of two parts. All statements previous to the statement, "READ (5,17) NPL" are involved in computing the pattern for a single element, while all statements after that statement calculate the pattern for the array (horizontal plane only, i.e. θ = 90 degrees) with amplitude and phase shading incorporated.

Dependent on the initial "NPL" code, the "DO" loop ending with statement number six (6) varies either angle θ or angle ϕ in one (1) degree increments.

The angle held constant (second entry on the third card of the input data deck) in consonance with the angle to be varied (dictated by the "NPL" code) specifies the plane in which the pattern is determined.

In the second part of the subroutine (amplitude and phase shading), the "DO" loop ending with statement number sixteen (16) is the coded version of the general expression for the amplitude and phase shading derived earlier for the horizontal plane.

<u>Called By:</u> MAIN

<u>Calls To:</u> HANK
POLPRT

POLPRT

<u>Purpose:</u> To control the plotting of the polar plot.

<u>Method:</u> This subroutine is the main subroutine in the polar plot package and is responsible for calling the various subroutines of the package.

The scale factor, S, must be changed according to the printer characteristics. The scale factor in this subroutine is set for ten, 10, characters per inch for the abscissa and eight, 8, characters per inch for the ordinate axis. Therefore S = 10./8.

After initializing DATAX, DATAY, and X, the input data, Y, is scanned to determine the normalizing factor. If this normalizing factor is less than 1.E-32, an error statement is printed and the plotting is aborted.

In the DO LOOP ending with statement 8, each line of the polar plot is printed after a call is made to PTPLOT to establish the polar grid information. The variable, DIM,

is used as a scaling factor for the polar plot. The value of 1.0 will cause all of input data to be plotted, however, if only the values less than one-half of the normalizing factor are of interest, then DIM can be set to .5. This will enlarge the center of the polar plot.

<u>Called By</u>: SEG
PATCH
DISK

<u>Calls To</u>: PTPLOT
SART

PTPLOT

<u>Purpose</u>: To establish the grid information for the polar plot.

<u>Method</u>: In the DO LOOP ending at statement 1 the alpha-numeric characters are transferred to ISYM in order to pass via COMMON to other subroutines. In the statements following statement 2, the equations for the plotted concentric circles are established. Below statement 7 the grid marks on the 090-270 axis are inserted.

<u>Called By</u>: POLPRT

<u>Calls To</u>: LINECK
NUMB

NUMB

Purpose: To place degree numbers on the polar plot.

Method: The current line which is being printed is passed to the subroutine in the calling argument. If this line contains degree numbers, these numbers are placed in the correct position by the IF statements.

Called By: PTPLOT

Calls To: NONE

LINECK

Purpose: To insert grid characters on the polar plot.

Method: The period character (ISM(2)) is inserted in the proper position in the statements above statement 4. In the statements after statement 4, the grid numbers labels are inserted on the horizontal axis.

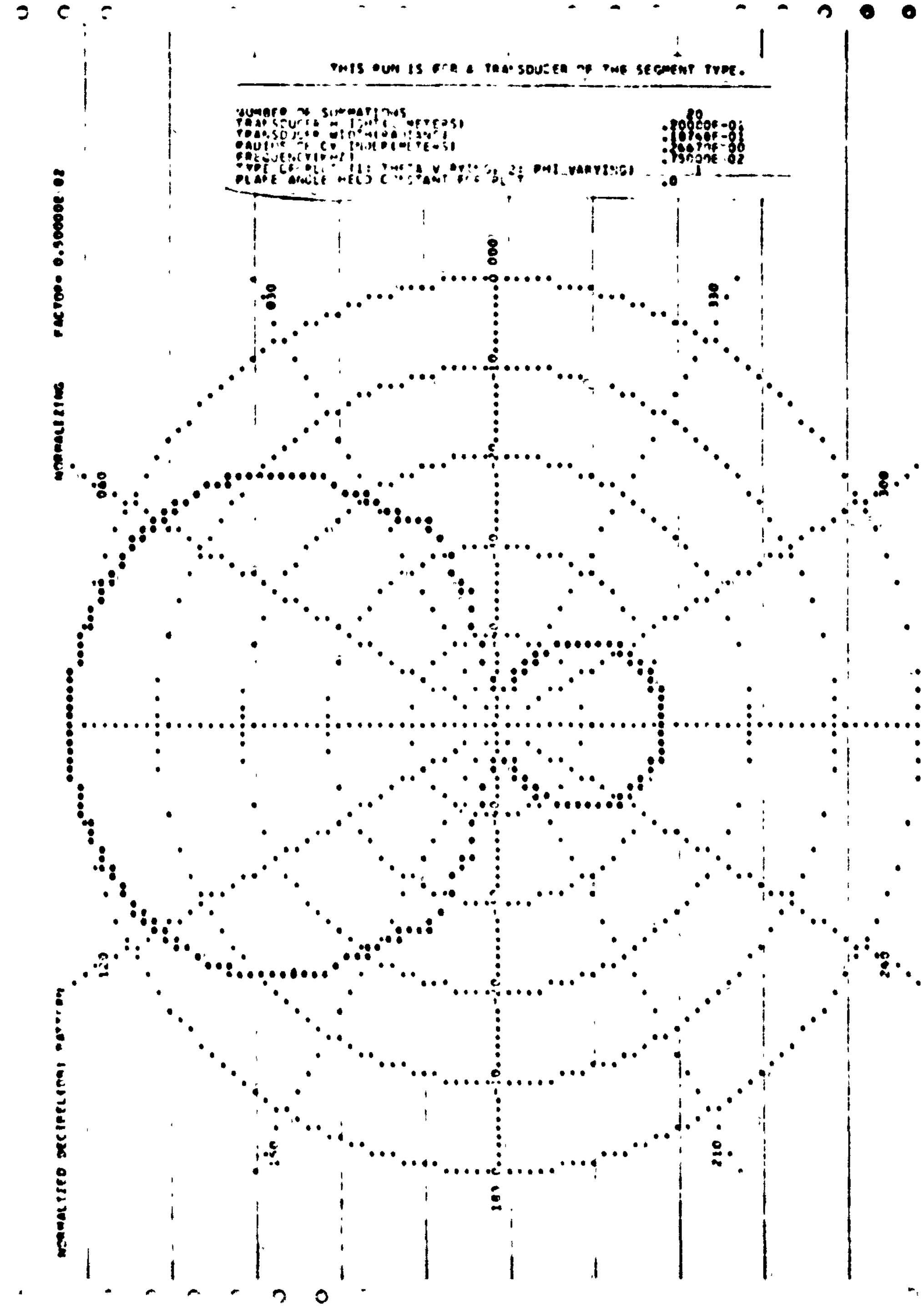
Called By: POLPLOT

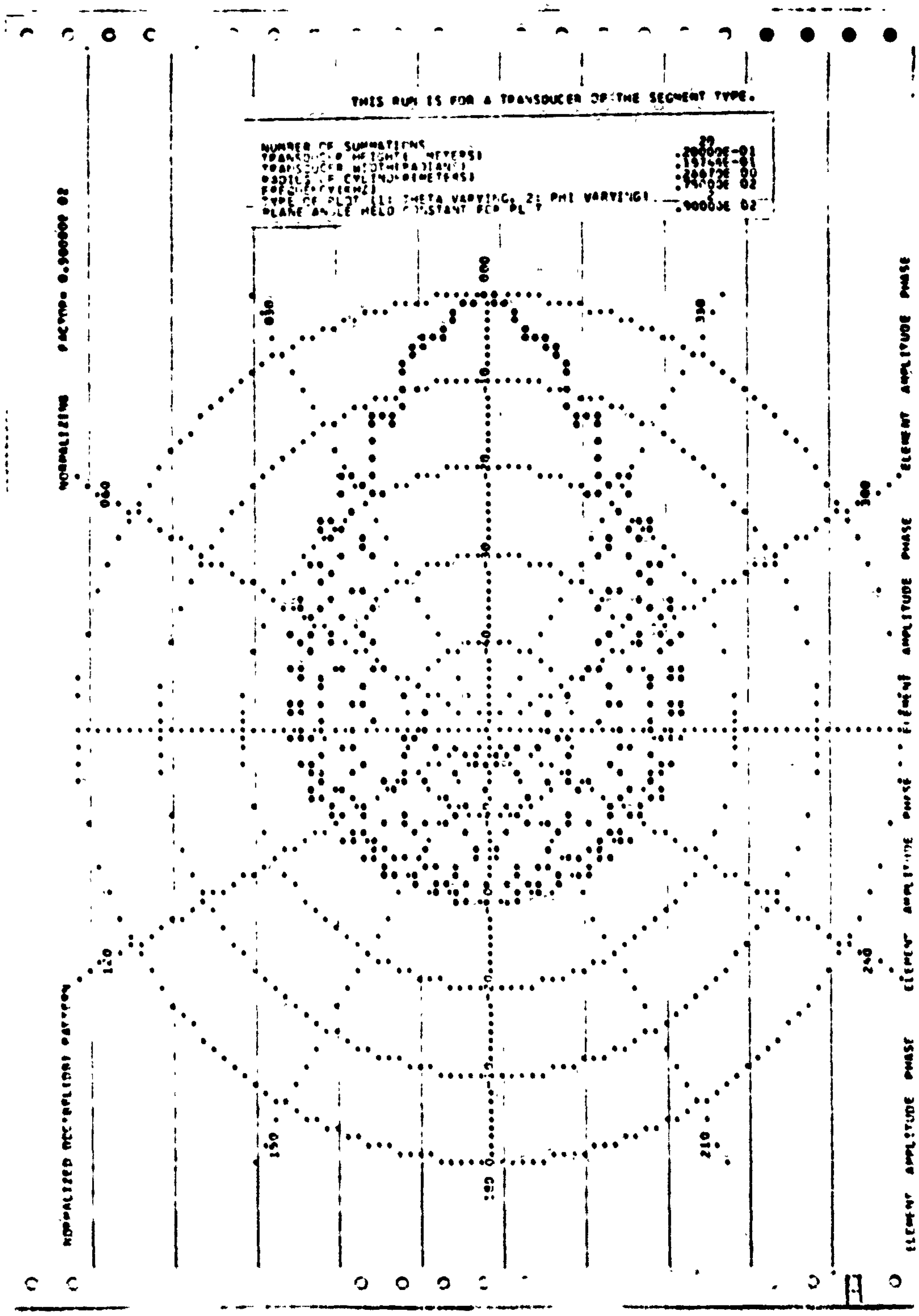
Calls To: NONE

APPENDIX C

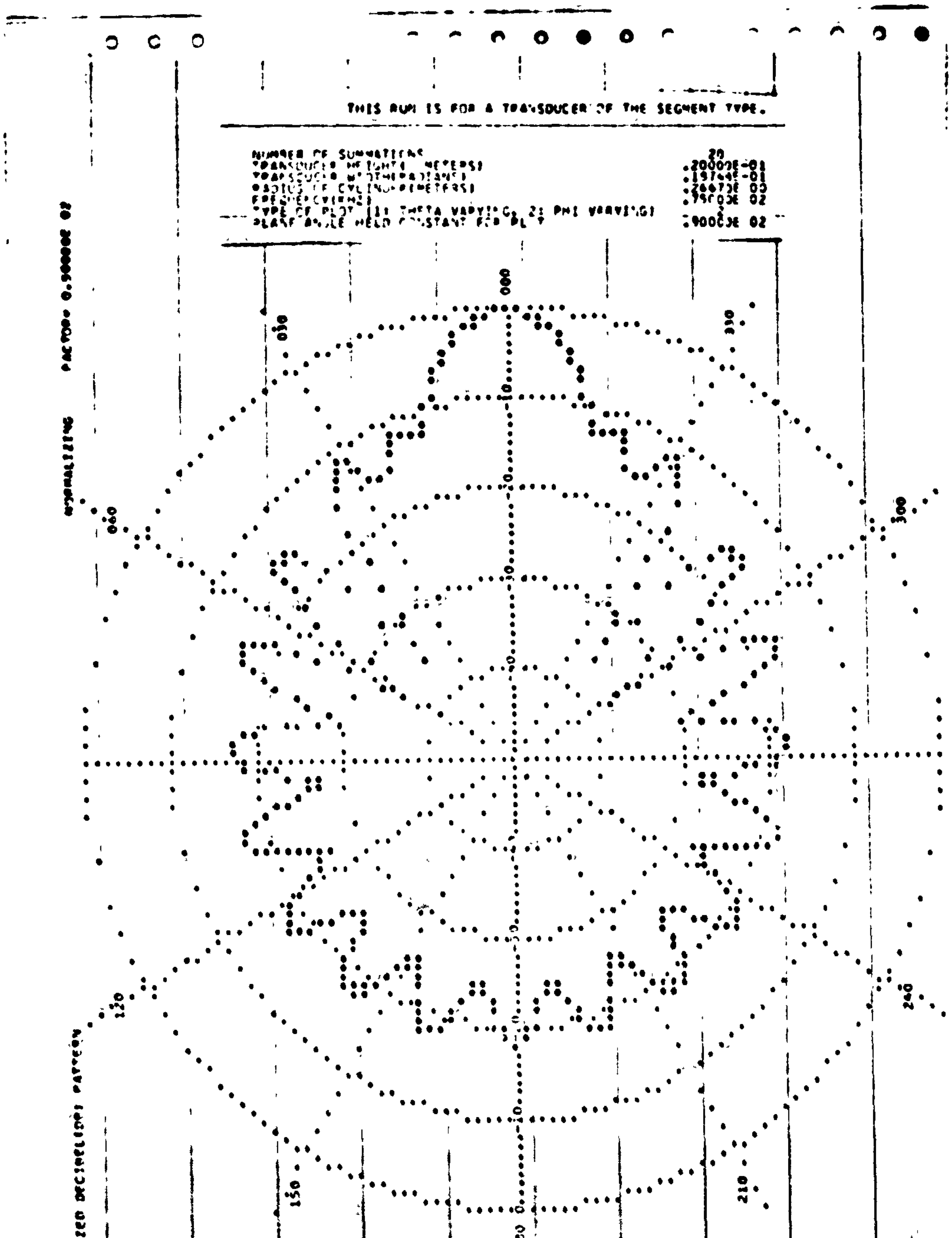
```
SEGEMENT
20        .02        .0187479   .2667     75.
 1        0.
SEGEMENT
20        .02        .0187479   .2667     75.
 2        90.
 2
  7
118       1.         0.
119       .8         0.
120       .3         0.
  1       .1         0.
  2       .3         0.
  3       .8         0.
  4       1.         0.
PATCH
20        .02        .0187479   .2667     75.
 1        0.
PATCH
20        .02        .0187479   .2667     75.
 2        90.
DISK
20        .025       .0135      .2667     75.
 1        0.
DISK
20        .025       .0135      .2667     75.
 2        90.
```

APPENDIX D

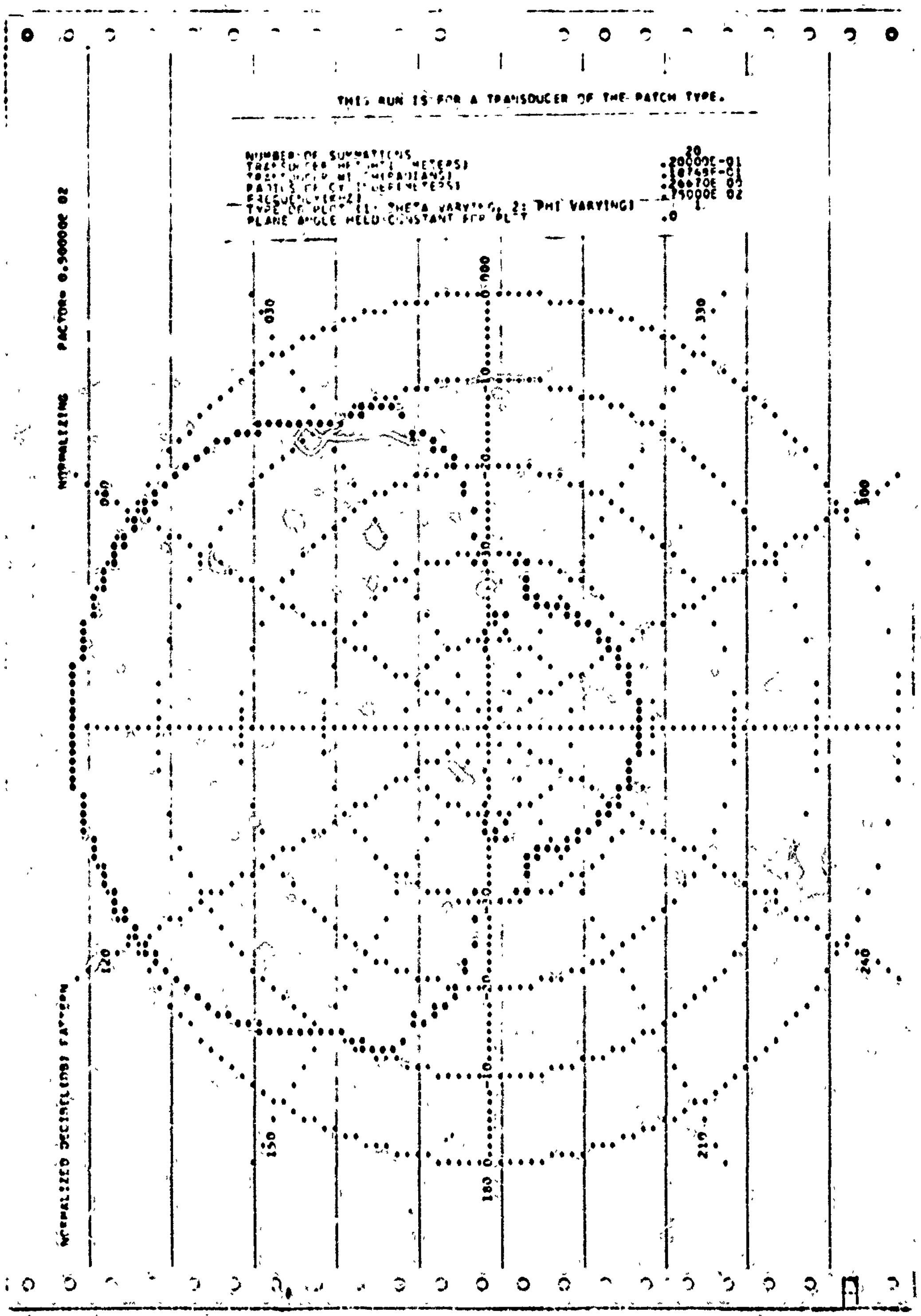




THIS RUN IS FOR A TRANSDUCER OF THE SEGMENT TYPE.
NUMBER OF SUMMATIONS
.90000E 02
000
030
060
120
150
180
210
240
300
330
NORMALIZED DECIBEL PATTERN
ELEMENT AMPLITUDE PHASE

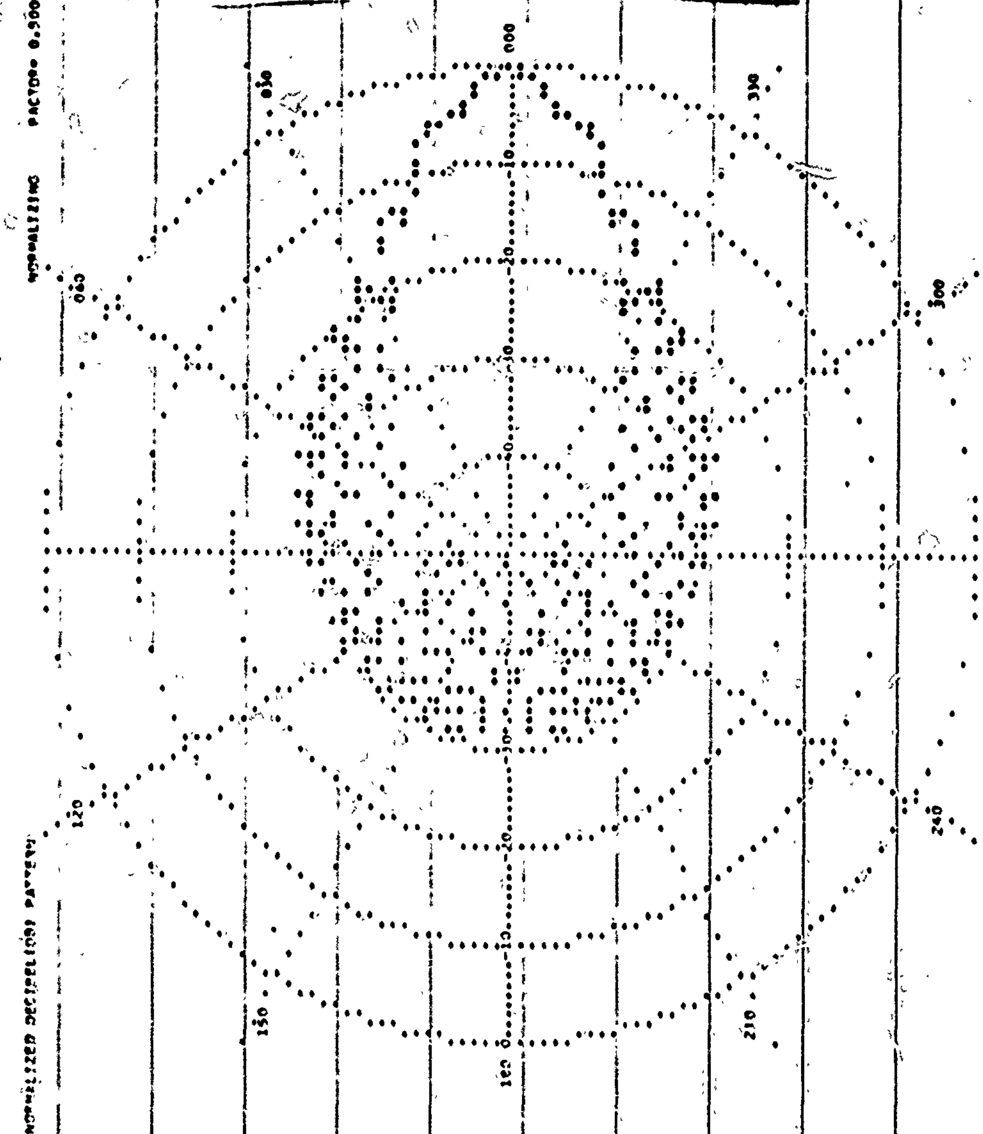


SEVEN(7) ELEMENTS SHADED AS PER APPENDIX C


THIS RUN IS FOR A TRANSDUCER OF THE PATCH TYPE.
NUMBER OF SUMMATIONS 20
.20000E-01
.75000E 02
PHI VARYING)
PLANE ANGLE HELD CONSTANT FOR PLOT .0
NORMALIZING FACTOR= 0.50000E 02
NORMALIZED DECIBEL(DB) PATTERN
000
030
330
060
300
120
240
150
210
180

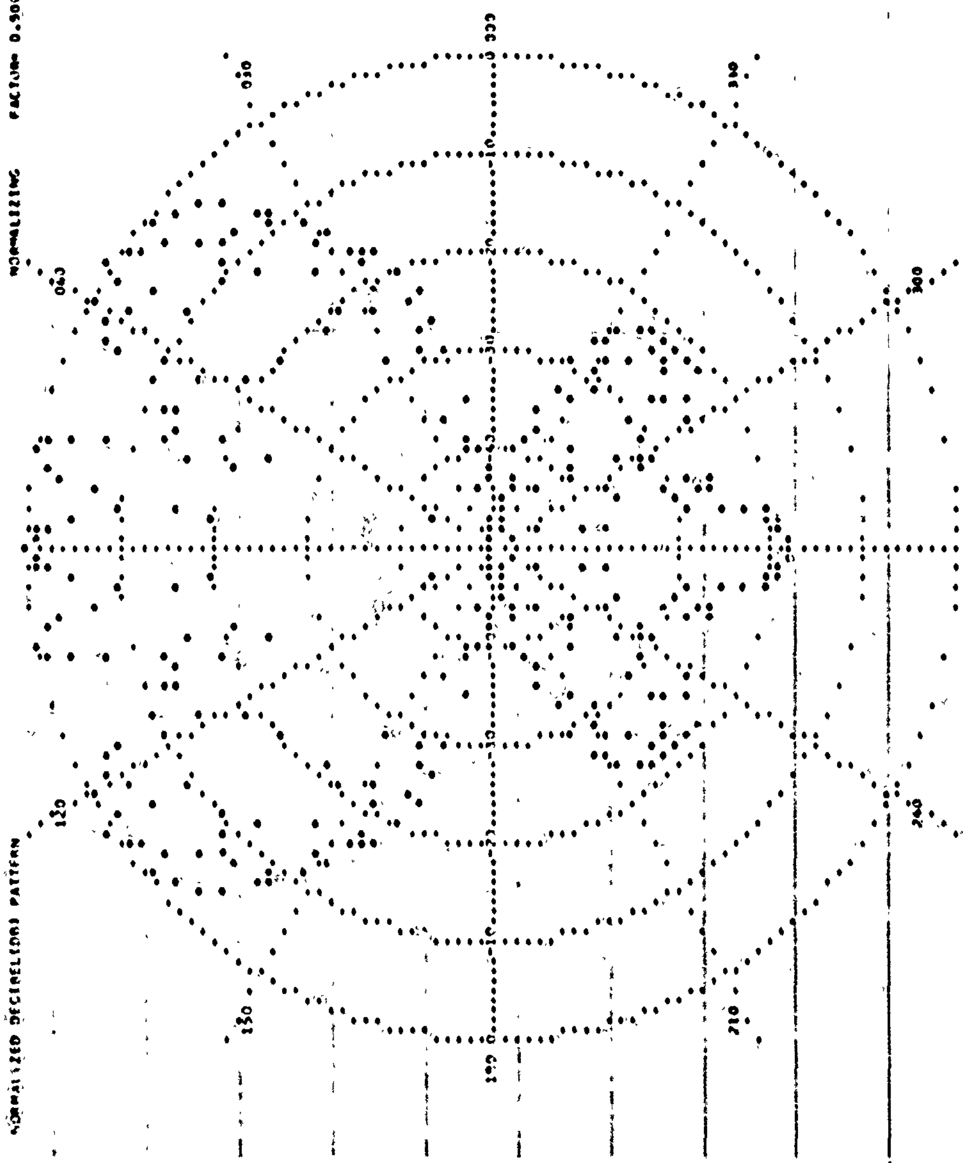
THIS RUN IS FOR A TRANSDUCER OF THE PATCH TYPE.

NUMBER OF SUMMATIONS	20
TRANSDUCER HEIGHT(METERS)	.20000E-01
TRANSDUCER WIDTH(RADIANS)	.18748E-01
RADIUS OF CYLINDER(METERS)	.26670E 00
FREQUENCY(KHZ)	.75000E 02
TYPE OF PLOT (1: THETA VARYING, 2: PHI VARYING)	2
PLANE ANGLE HELD CONSTANT FOR PLOT	.90000E 02



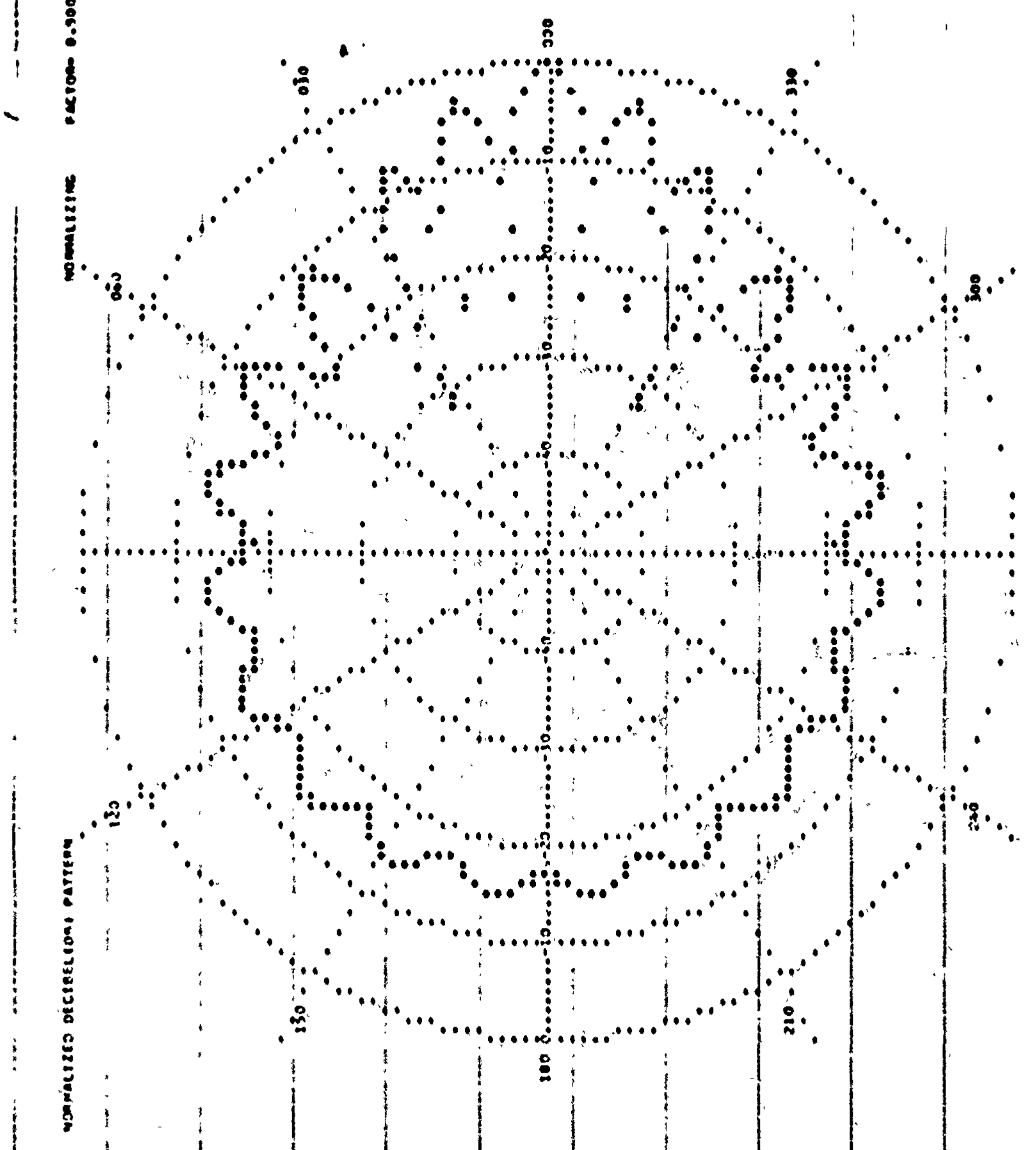
THIS RUN IS FOR A TRANSDUCER OF THE DISK TYPE.

NUMBER OF SUMMATIONS	[illegible]
OUTER RADIUS OF DISK(METERS)	.25000E-01
INNER RADIUS OF DISK(METERS)	.13500E-01
RADIUS OF CYLINDER(METERS)	[illegible]
FREQUENCY(HZ)	[illegible]
TYPE OF PLOT (1) THETA VARYING, 2) PHI VARYING)	1
PLANE ANGLE HELD CONSTANT FOR PLOT	.0



THIS RUN IS FOR A TRANSDUCER OF THE DISK TYPE.

NUMBER OF SUMMATIONS	22
OUTER RADIUS OF DISK(METERS)	.25000E-01
INNER RADIUS OF DISK(METERS)	.13500E-01
RADIUS OF CYLINDER(METERS)	.26670E 03
FREQUENCY(KHZ)	.75000E 02
TYPE OF PLOT (1: THETA VARYING, 2: PHI VARYING)	2
PLANE ANGLE HELD CONSTANT FOR PLOT	.90000E 02



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2. Morse, P.M., Vibration and Sound, McGraw-Hill, 1948.

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